The influence of spectral bandwidth and shape on wave breaking onset

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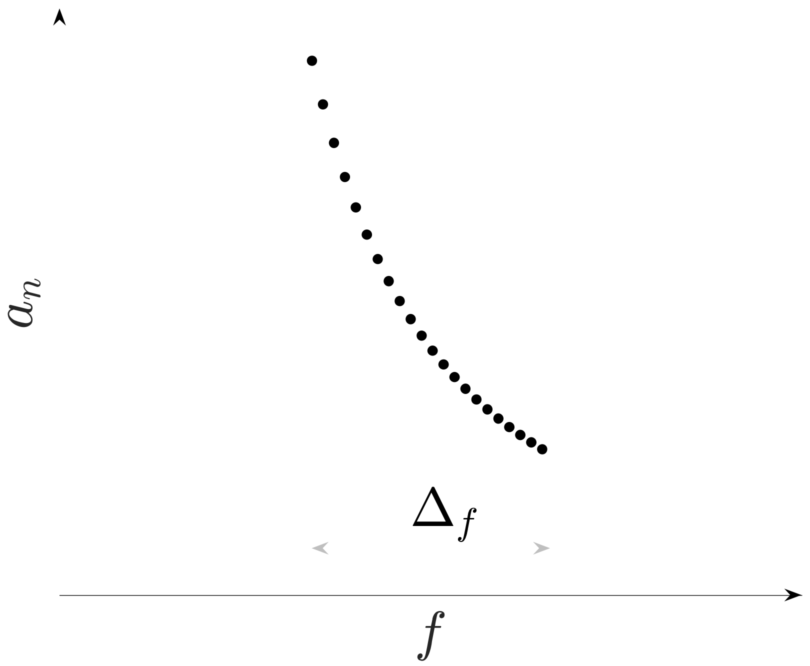
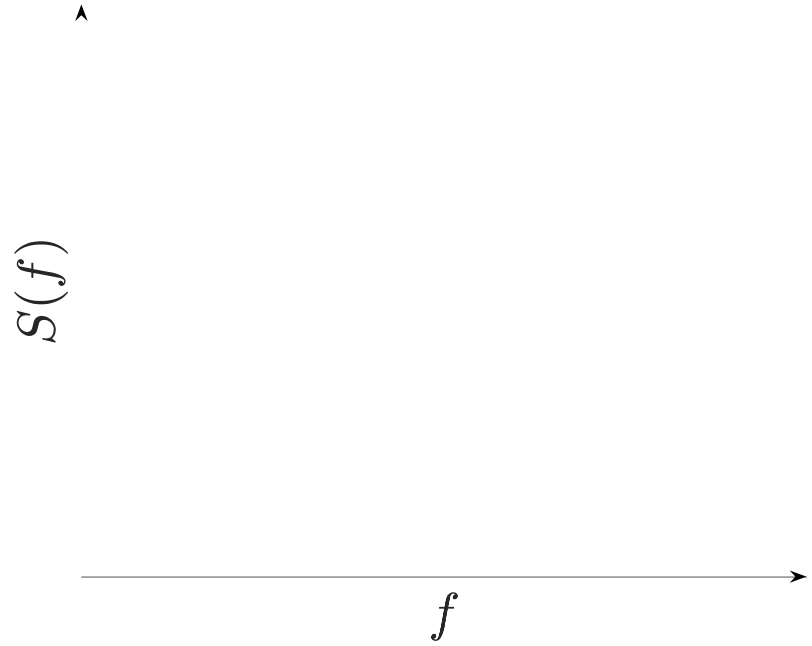
Frequency bandwidth (2D)



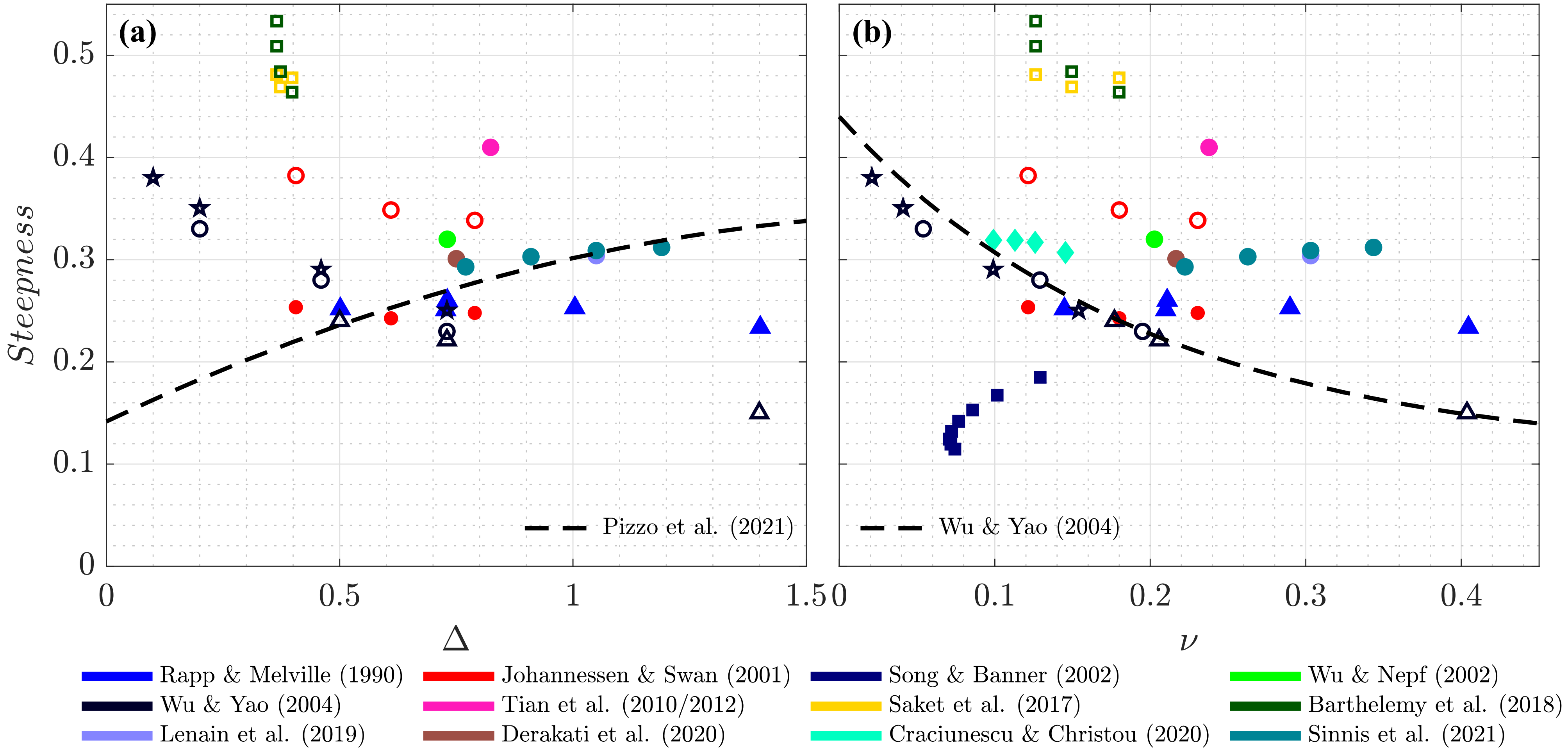


$$\Delta = \Delta_f / f_0$$

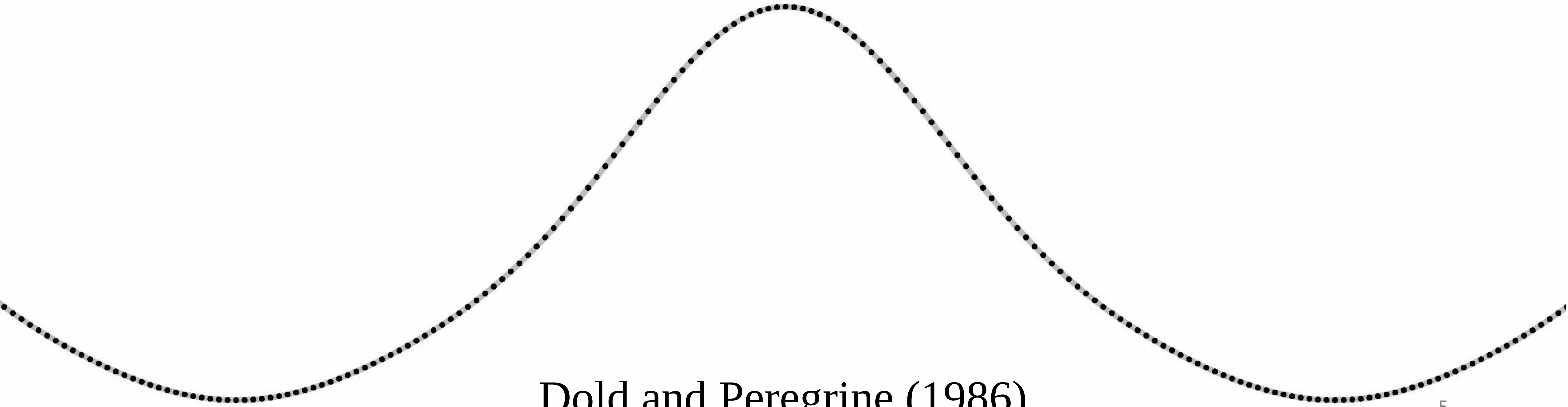
$$\nu = \sqrt{\frac{m_0 m_2}{m_1^2} - 1}\ \text{, where } m_n = \int s(f) f^n df$$

Wave breaking + Bandwidth

Onset Intensity Dissipation Mixing

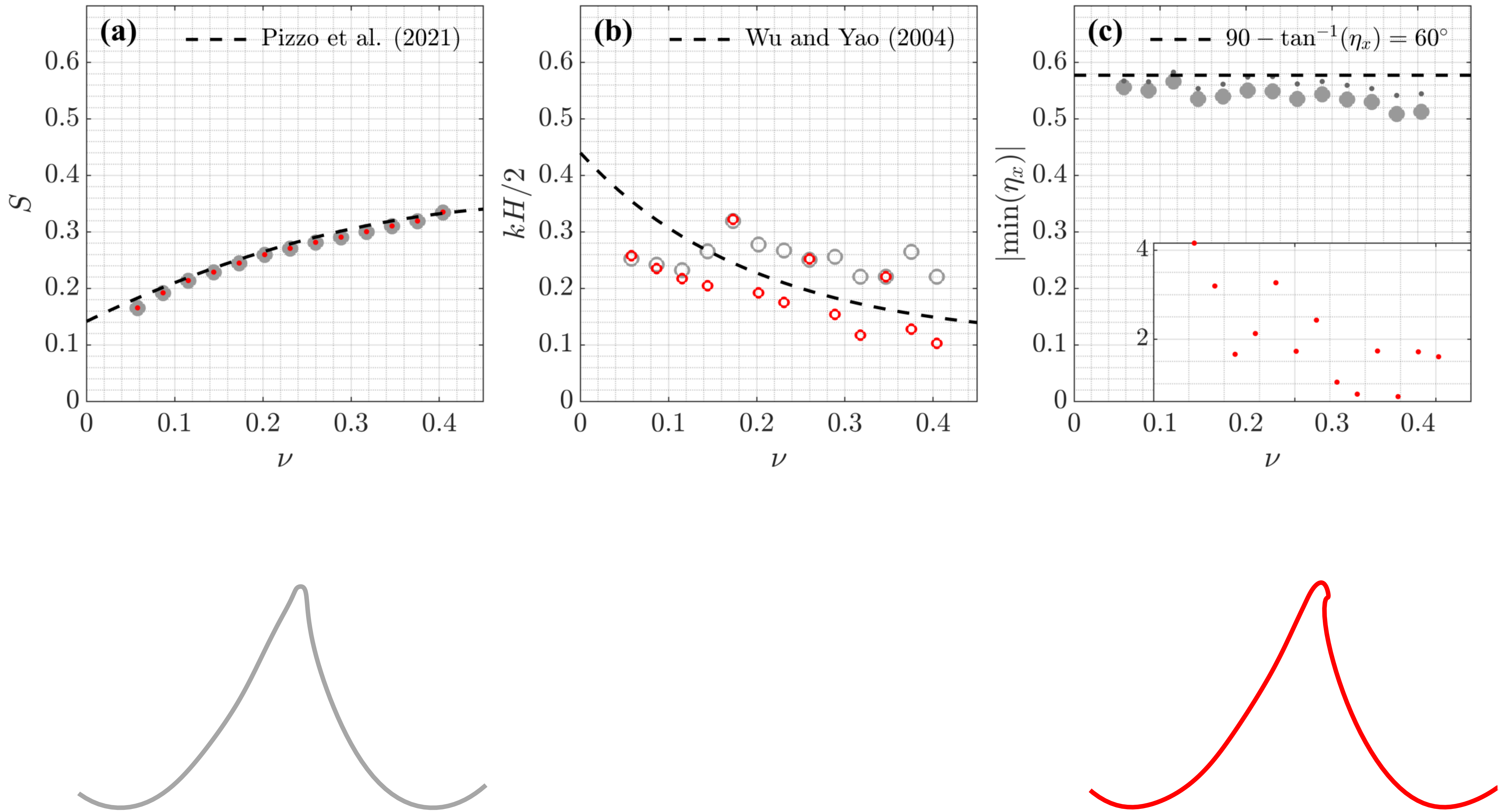

(a)
(b)
Steepness
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
0
0.1
0.2
0.3
0.4
Δ
ν
Pizzo et al. (2021)
Wu & Yao (2004)
Rapp & Melville (1990)
Johannessen & Swan (2001)
Song & Banner (2002)
Wu & Nepf (2002)
Wu & Yao (2004)
Tian et al. (2010/2012)
Saket et al. (2017)
Barthelemy et al. (2018)
Lenain et al. (2019)
Derakati et al. (2020)
Craciunescu & Christou (2020)
Sinnis et al. (2021)

Nonlinear Potential-flow Simulations

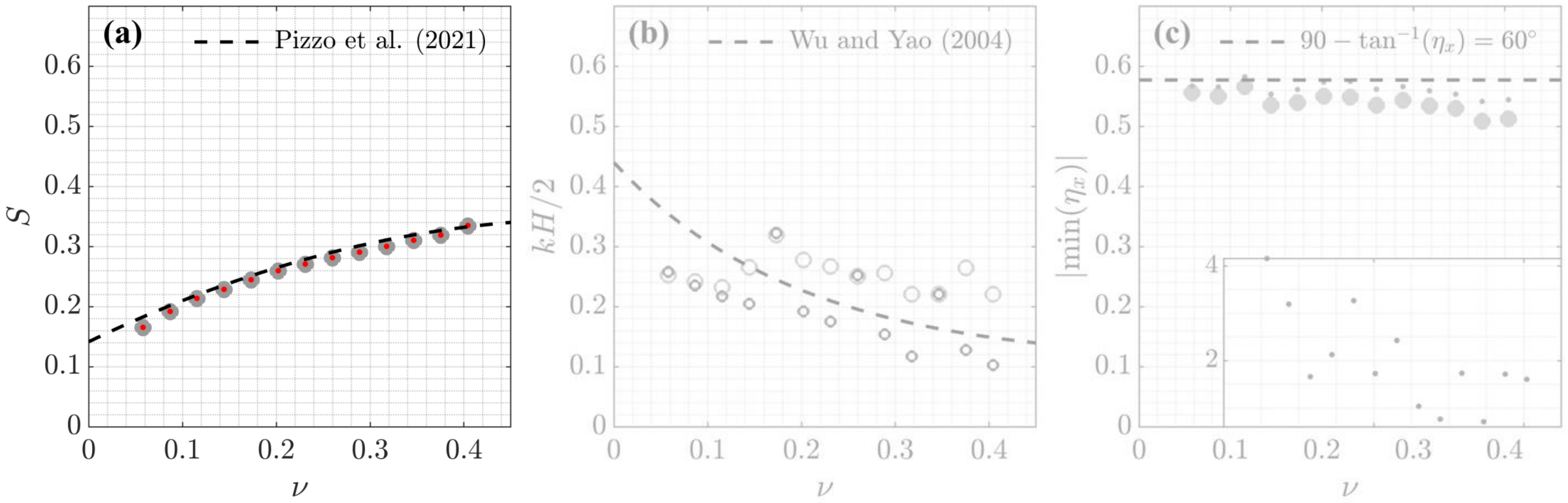


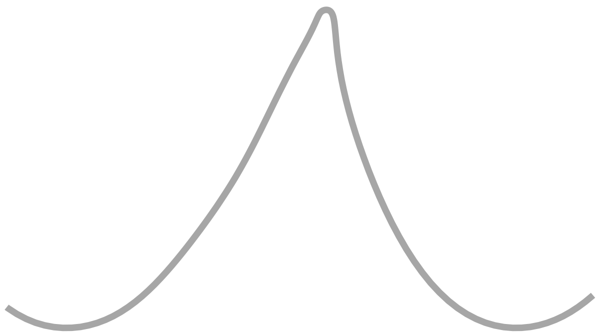
Dold and Peregrine (1986)

Global steepness Local steepness Local slope

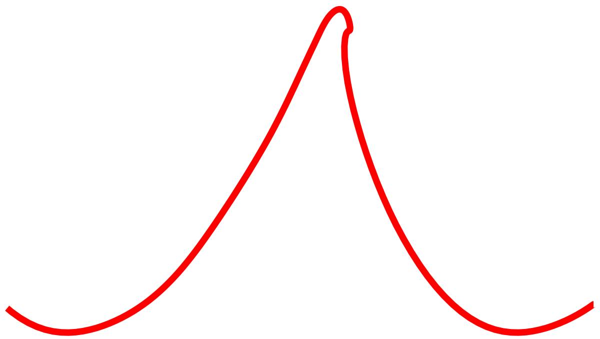


Global steepness Local steepness Local slope

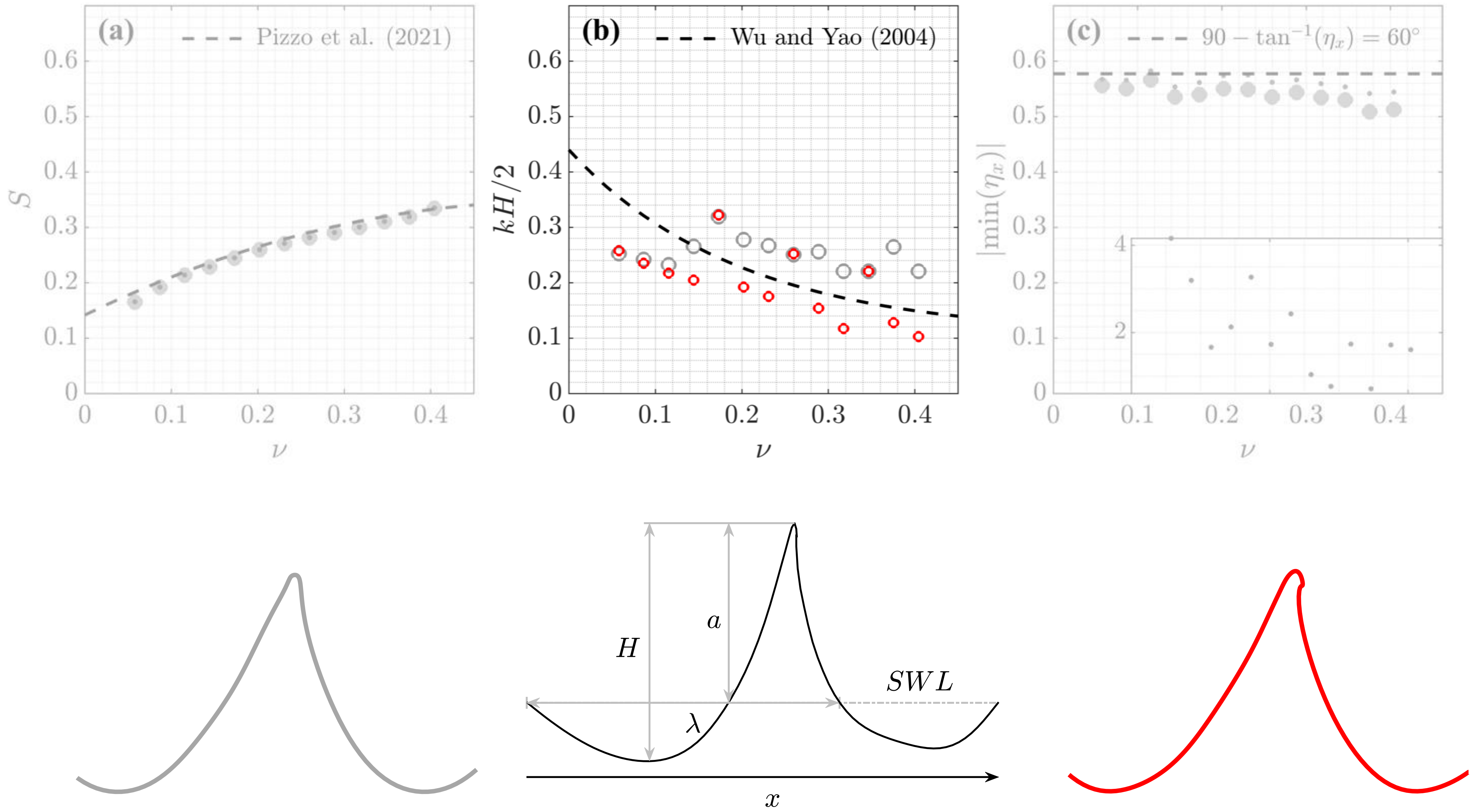




$$S = \sum a_n k_n \equiv k_c \sum a_n$$



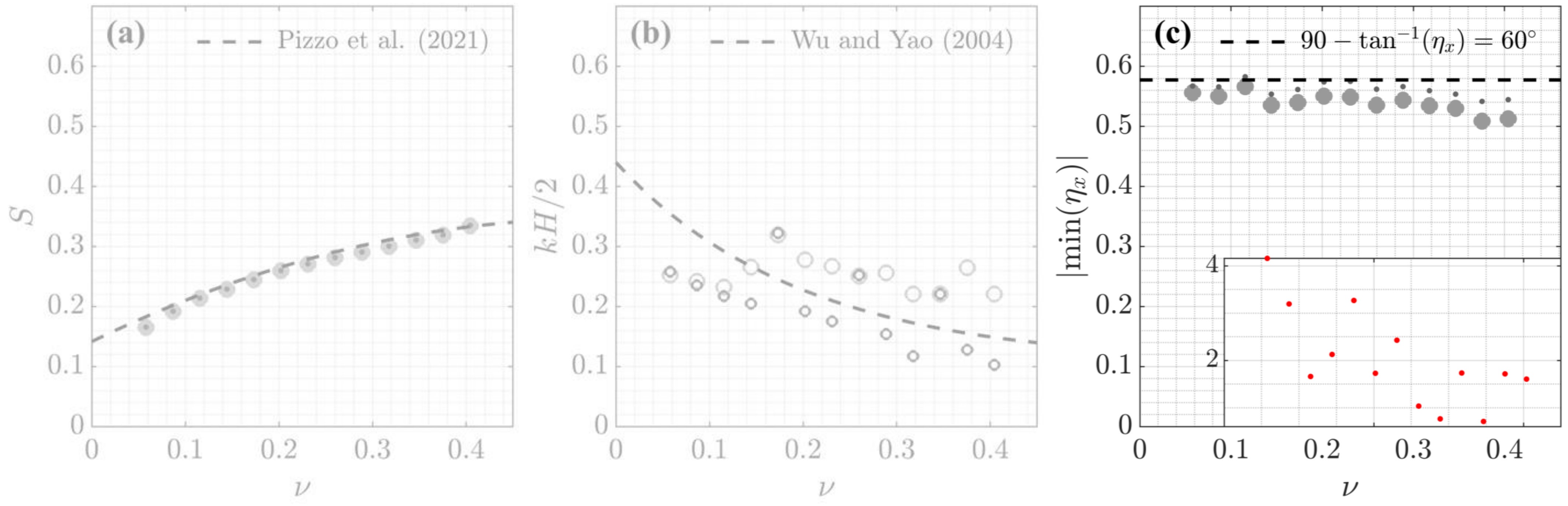
Global steepness Local steepness Local slope

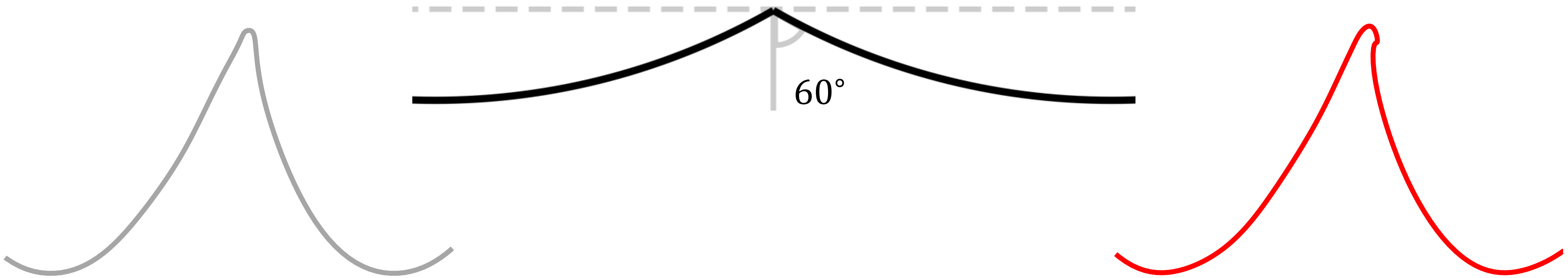


Global steepness

Local steepness

Local slope



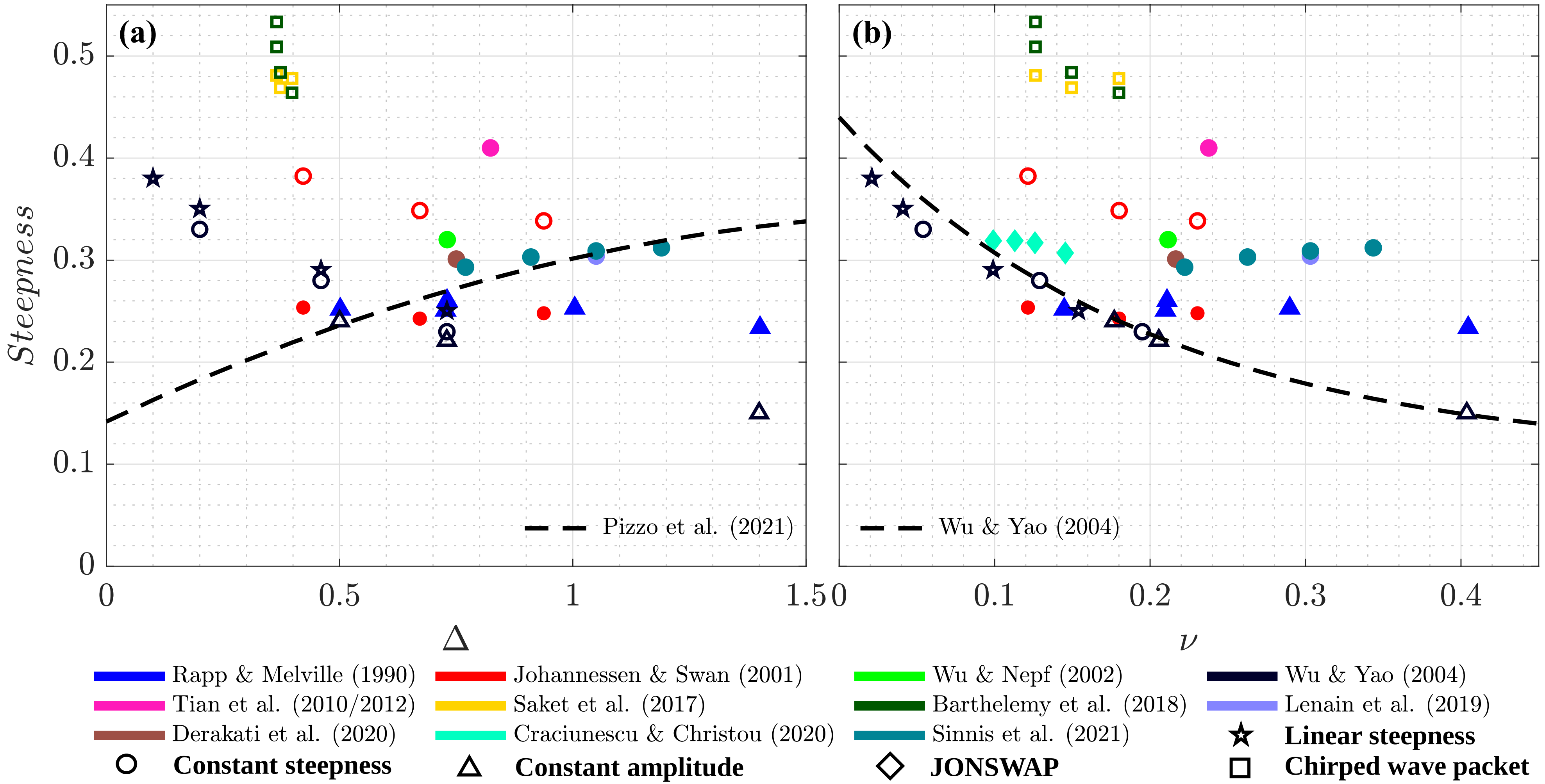


$As\ v\ increases\ for\ maximally\ steep\ waves$:

$Global\ steepness \uparrow$

$Local\ steepness\ \downarrow$

$Local\ slope \approx constant$



(a)
(b)
Steepness
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
0
0.1
0.2
0.3
0.4
Δ
ν
Pizzo et al. (2021)
Wu & Yao (2004)
Rapp & Melville (1990)
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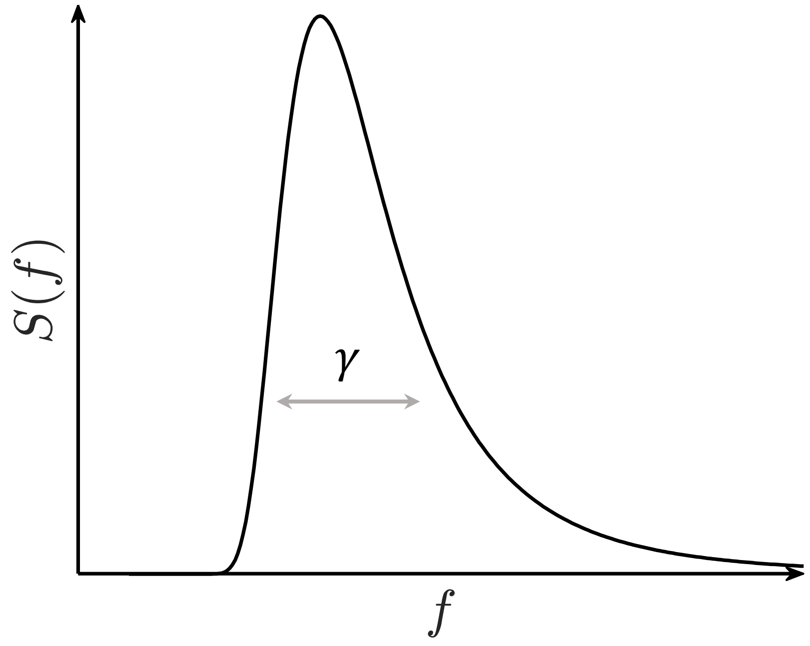
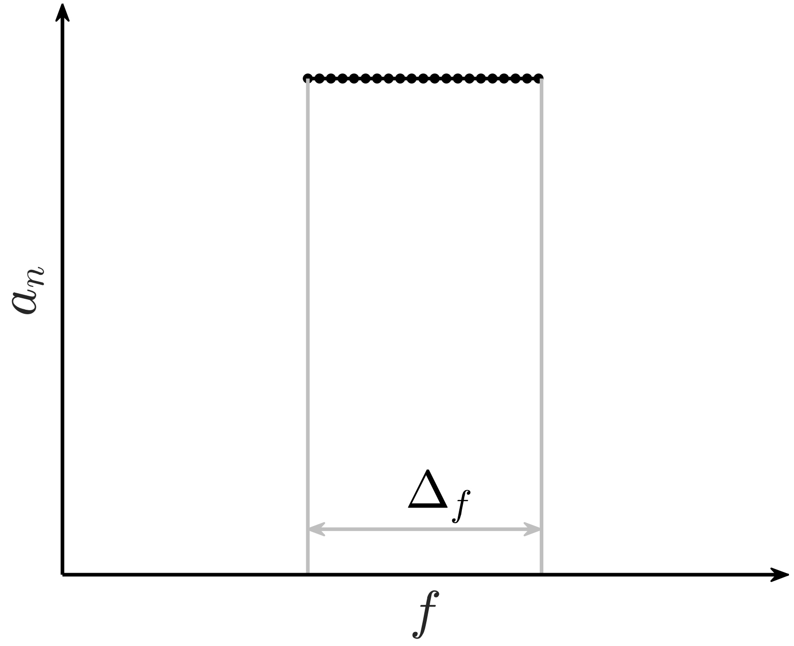
Lenain et al. (2019)
Derakati et al. (2020)
Craciunescu & Christou (2020)
Sinnis et al. (2021)
Linear steepness
Constant steepness
Constant amplitude
JONSWAP
Chirped wave packet

JONSWAP

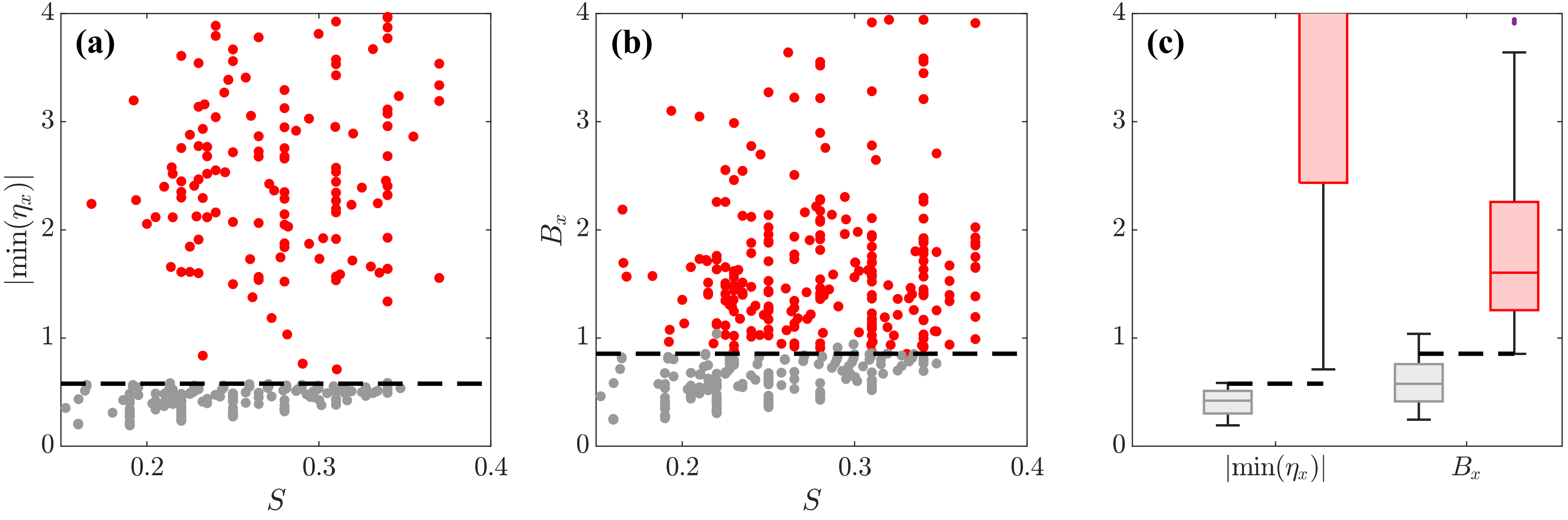


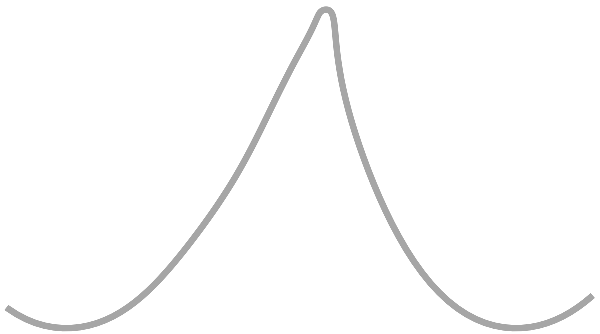
$\nu \propto \gamma$

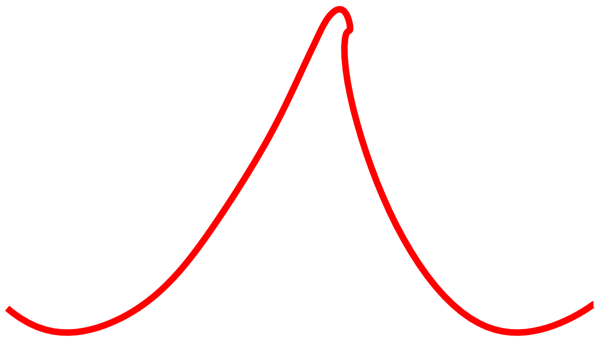
Constant amplitude



$\nu \propto \Delta$


(a)
(b)
(c)
$|\min(\eta_x)|$
B_x
S
0
1
2
3
4
0.2
0.3
0.4

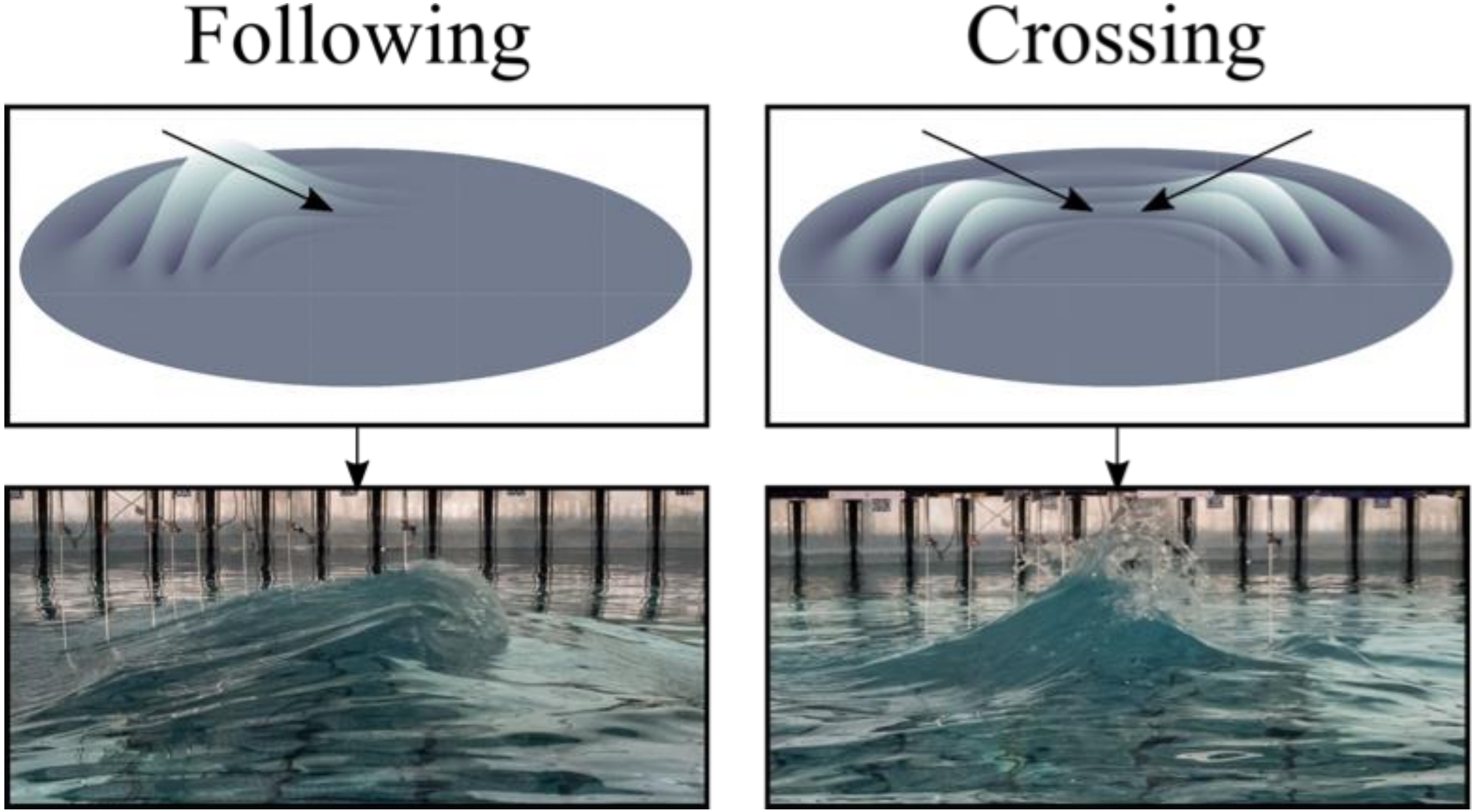




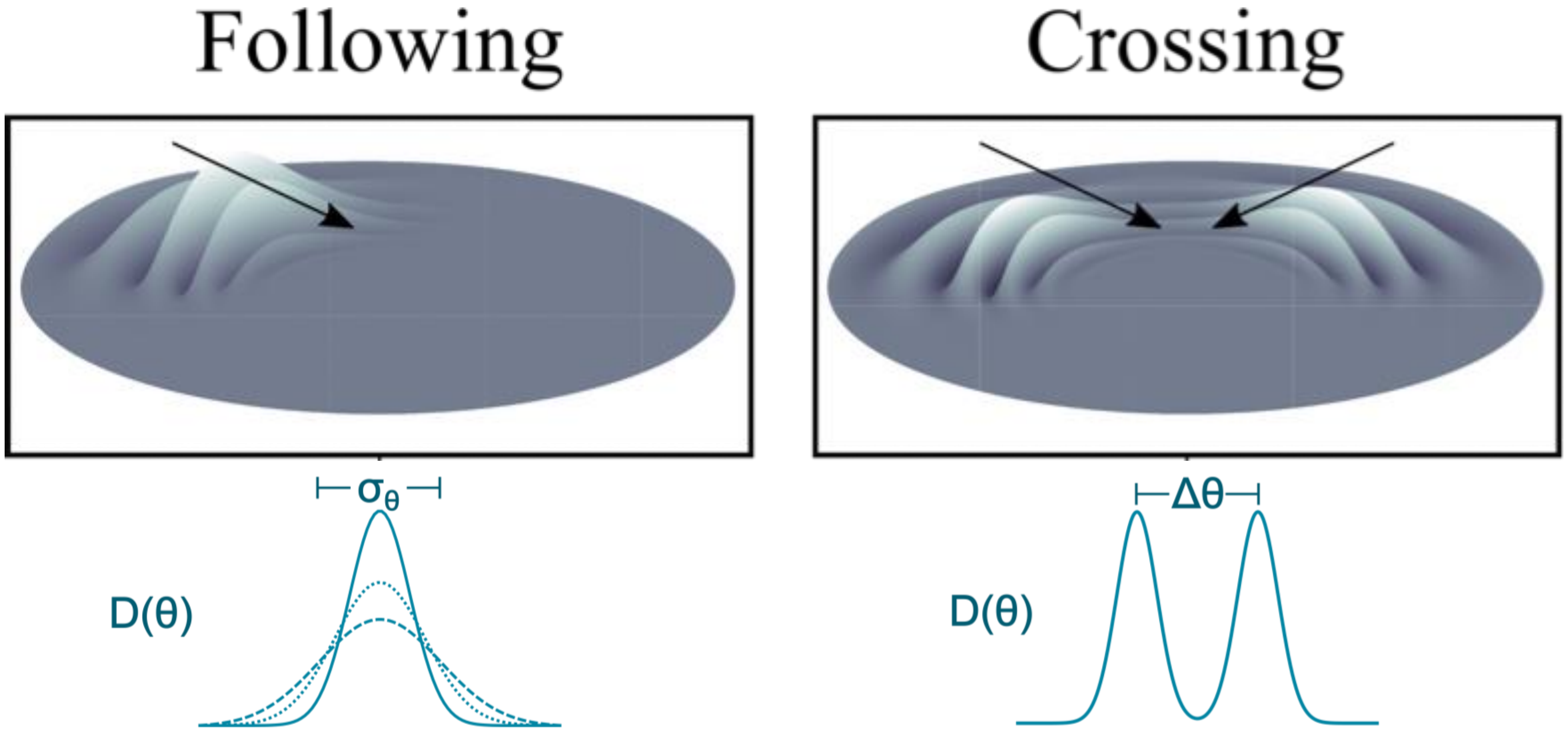
Frequency bandwidth & spectral shape (2D):

- Local steepness $kH/2$
  - Breaking onset reduces as function of bandwidth
  - Local steepness is a poor indicator of breaking onset
- Global steepness S
  - Breaking onset increases as function of bandwidth, and varies with shape
  - Global steepness is a good indicator of breaking onset
- Local slope
  - Breaking onset appears to occur at $\eta_x = 0.5774$, for all spectra and bandwidths

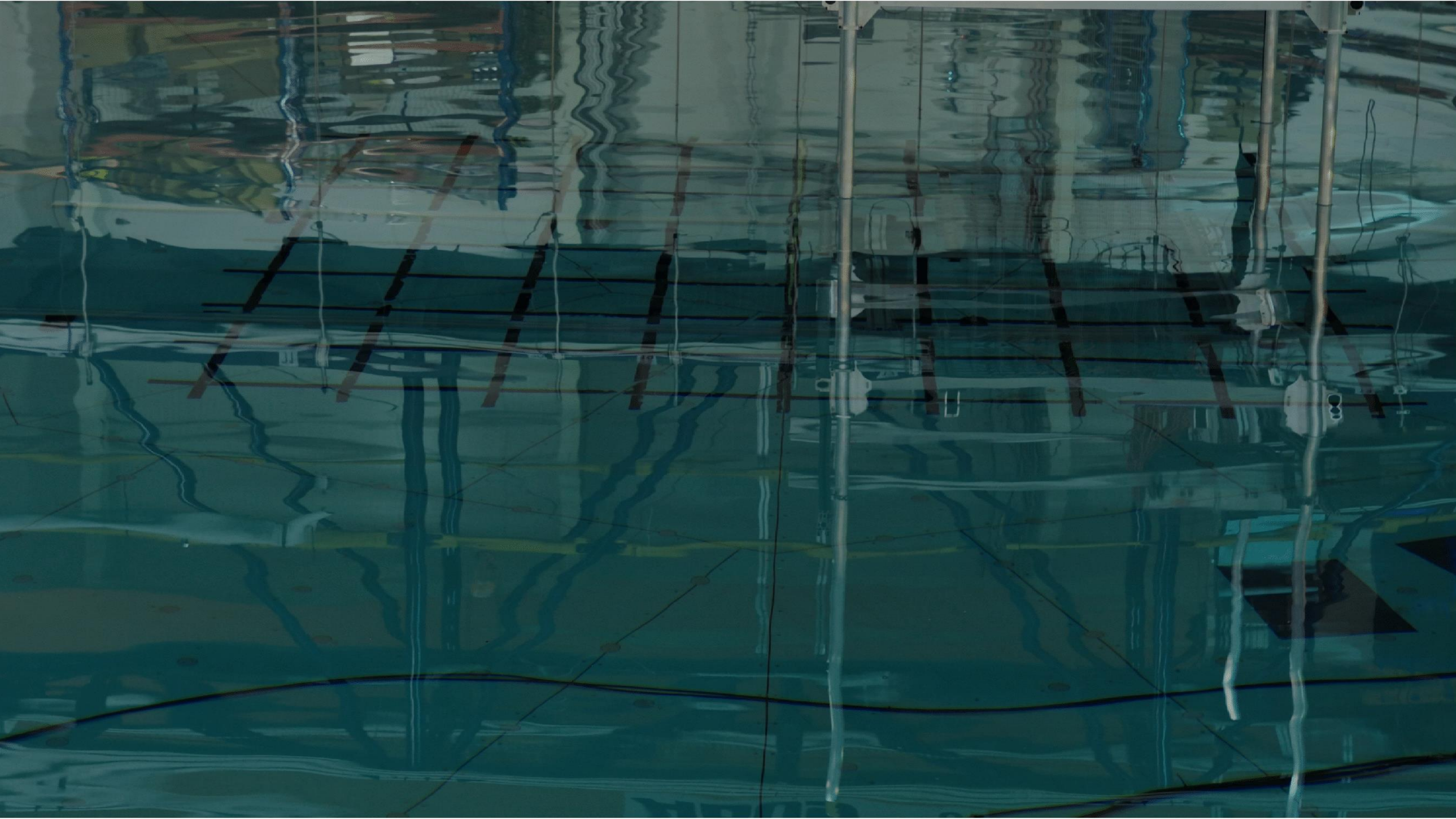
Directional bandwidth (3D)

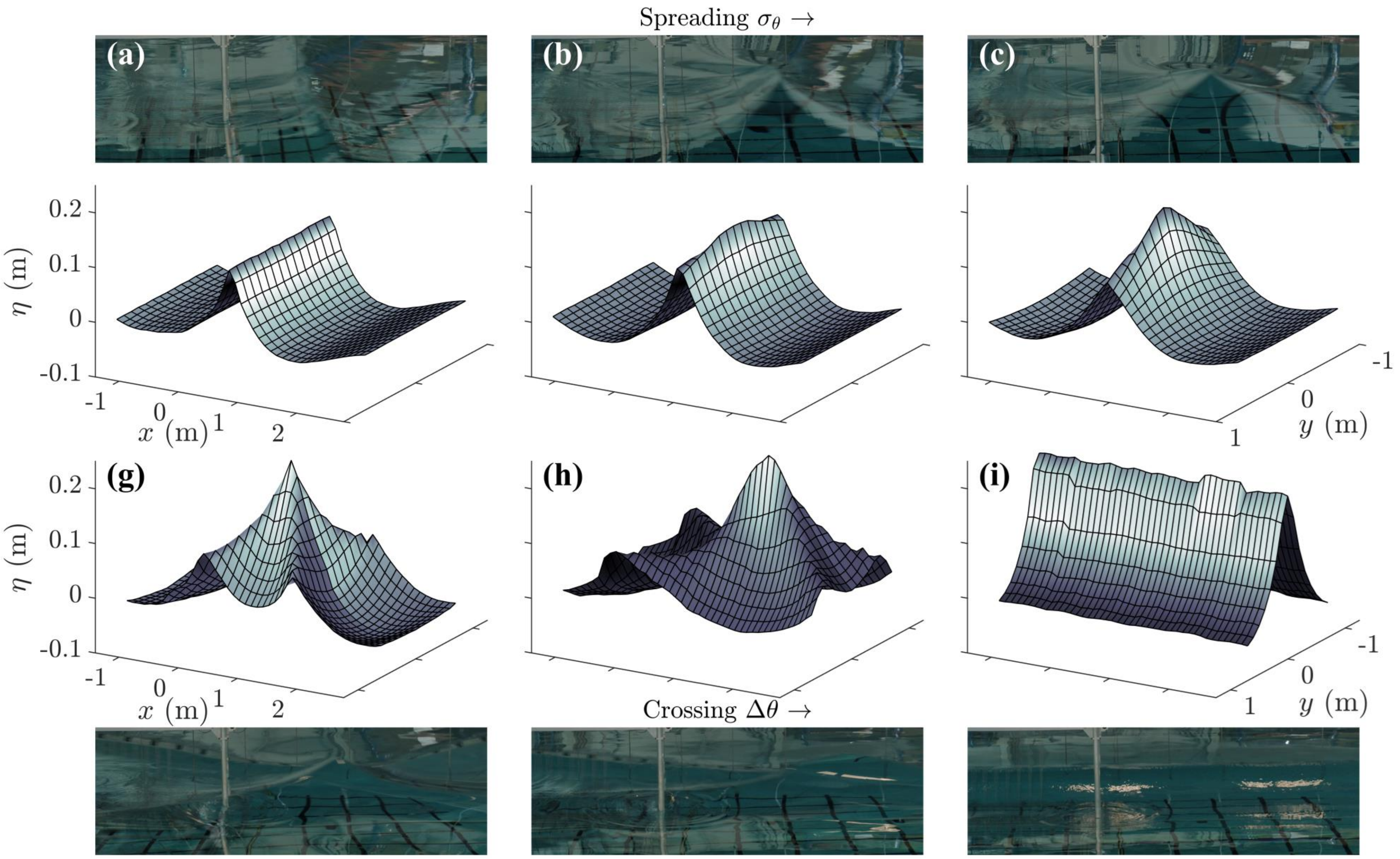


Directional bandwidth (3D)



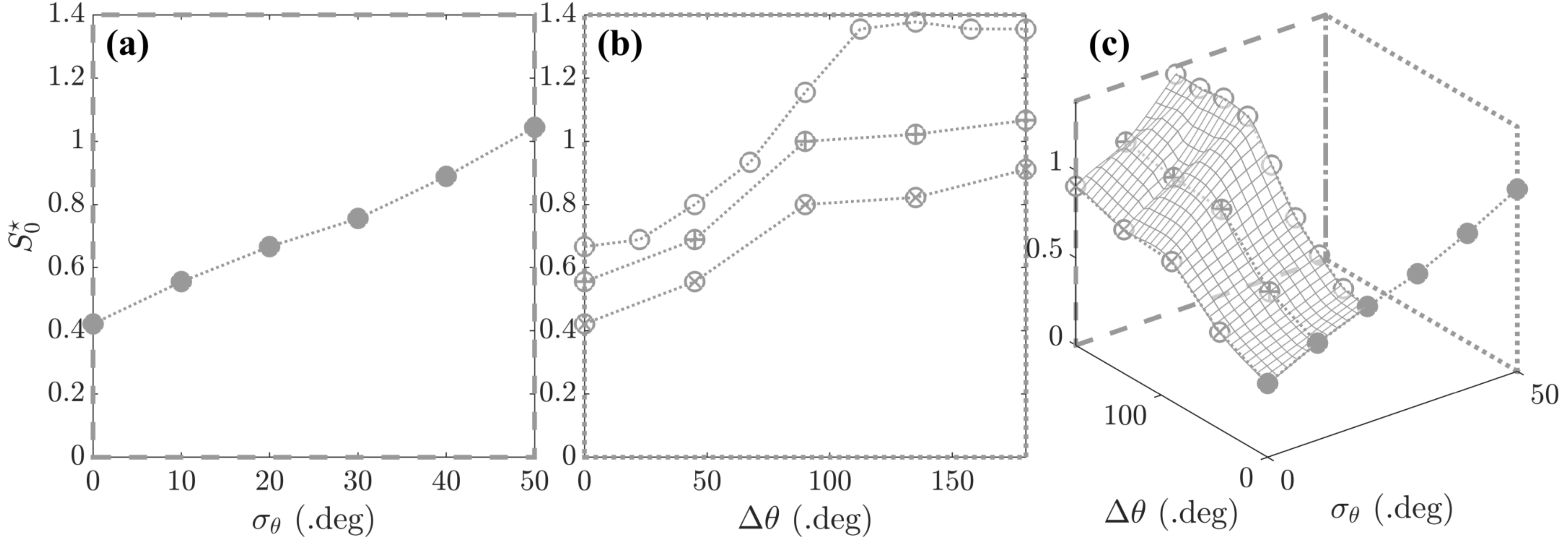
σ_θ (.deg)	$\Delta\theta$ (.deg)
0,10,20†,30,40,50	0
0	45,90,135,180
10	45,90,135,180
20	22.5,45†,67.5,90†,112.5,135†,157.5,180





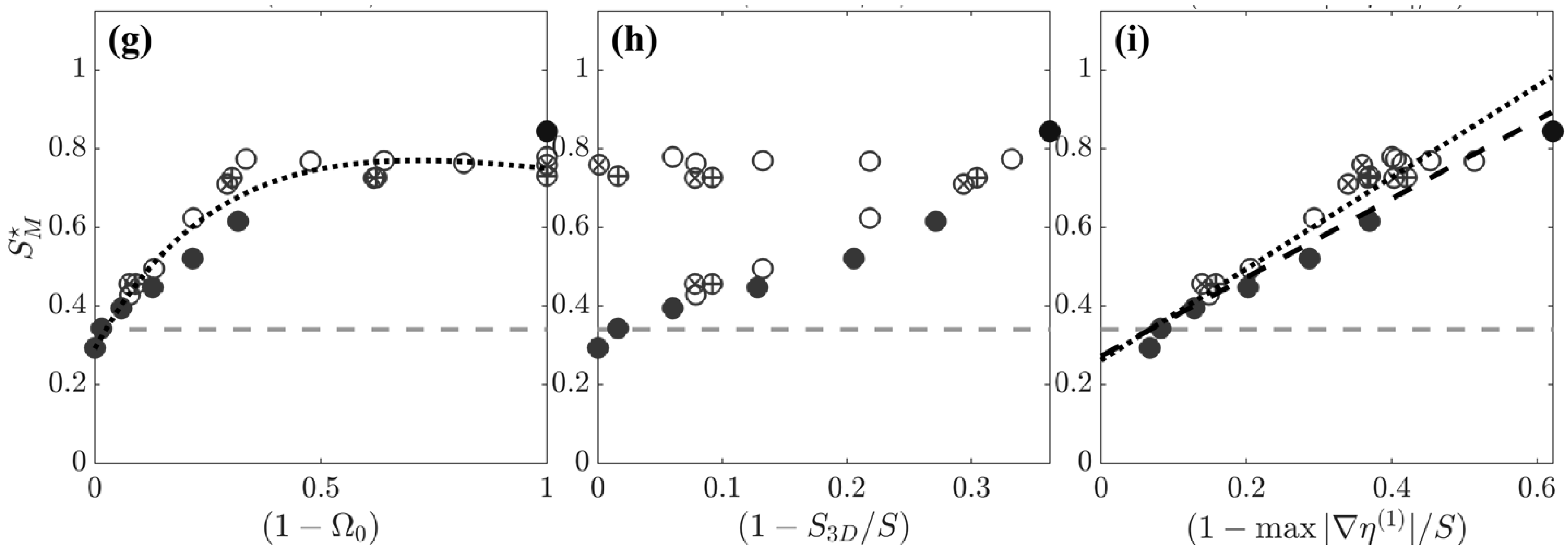
Spreading σ_θ →
(a)
(b)
(c)
η (m)
0.2
0.1
0
-0.1
-1
0
1
2
x (m)
y (m)
(g)
(h)
(i)
Crossing $\Delta\theta$ →

Directional breaking threshold



	σ_θ (.deg)	$\Delta\theta$ (.deg)
●	0,10,20†,30,40,50	0
⊗	0	45,90,135,180
⊕	10	45,90,135,180
○	20	22.5,45†,67.5,90†,112.5,135†,157.5,180

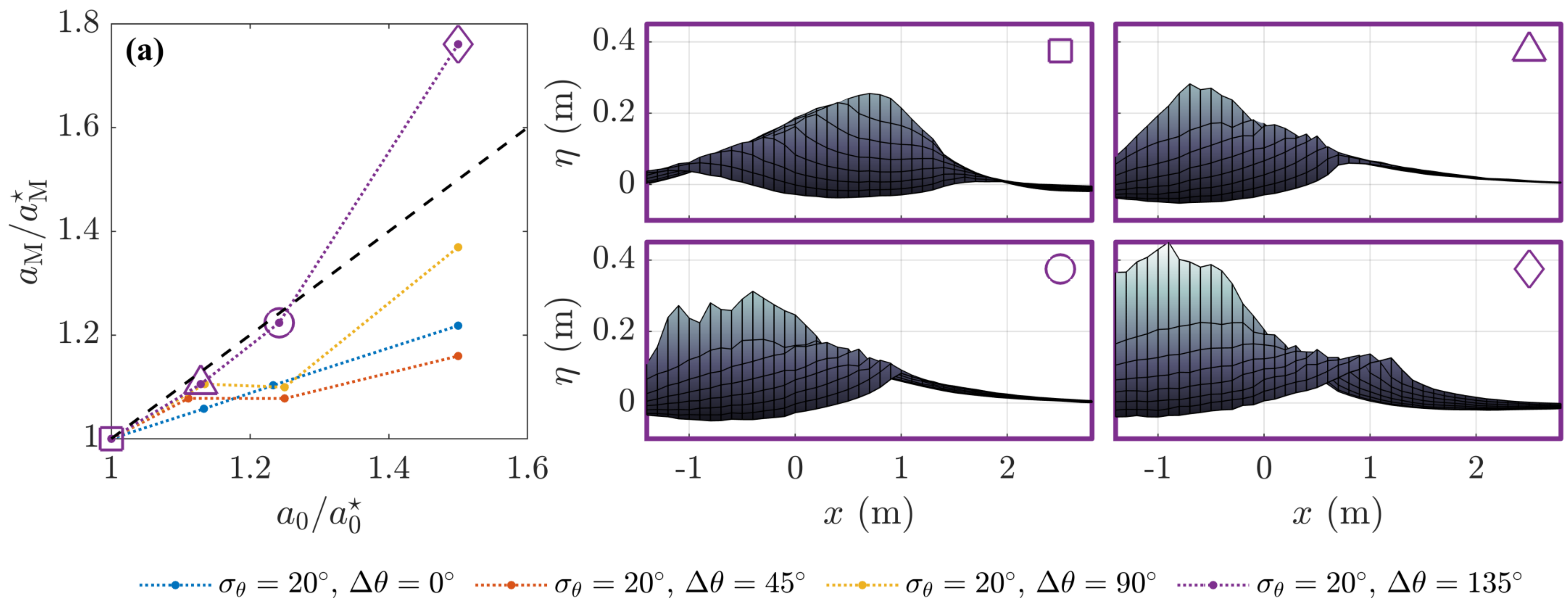
Directional breaking threshold



	σ_θ (.deg)	$\Delta\theta$ (.deg)
●	0,10,20†,30,40,50	0
⊗	0	45,90,135,180
⊕	10	45,90,135,180
○	20	22.5,45†,67.5,90†,112.5,135†,157.5,180

D

Post breaking behavior



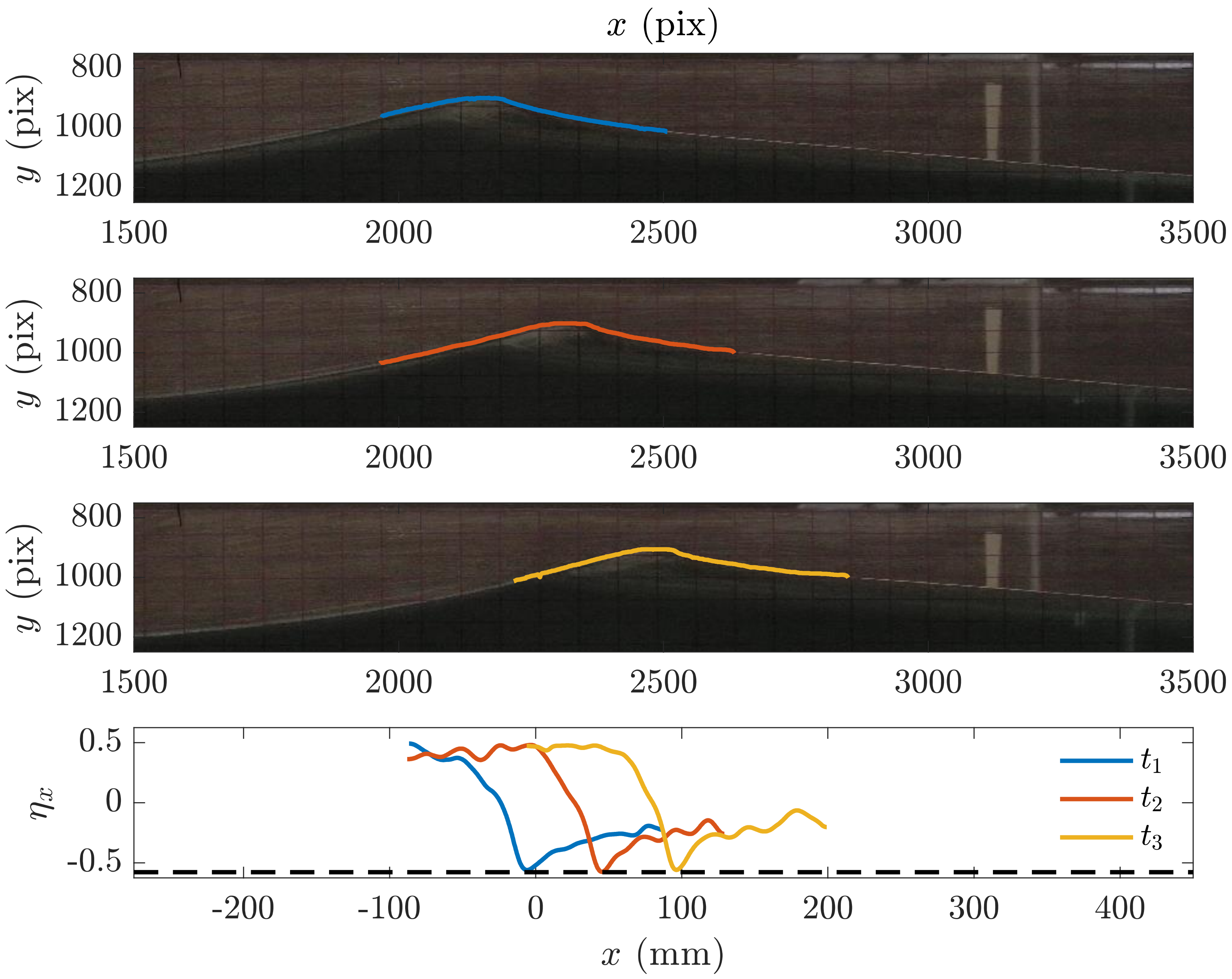
Directional bandwidth (3D):

- Global steepness
  - Breaking onset increases as function of directional bandwidth by as much as %100
  - How breaking onset steepness varies can be parameterized using a single parameter measure of spreading
- Post breaking behavior
  - Breaking for spread and crossing waves does not limit wave crest amplitude

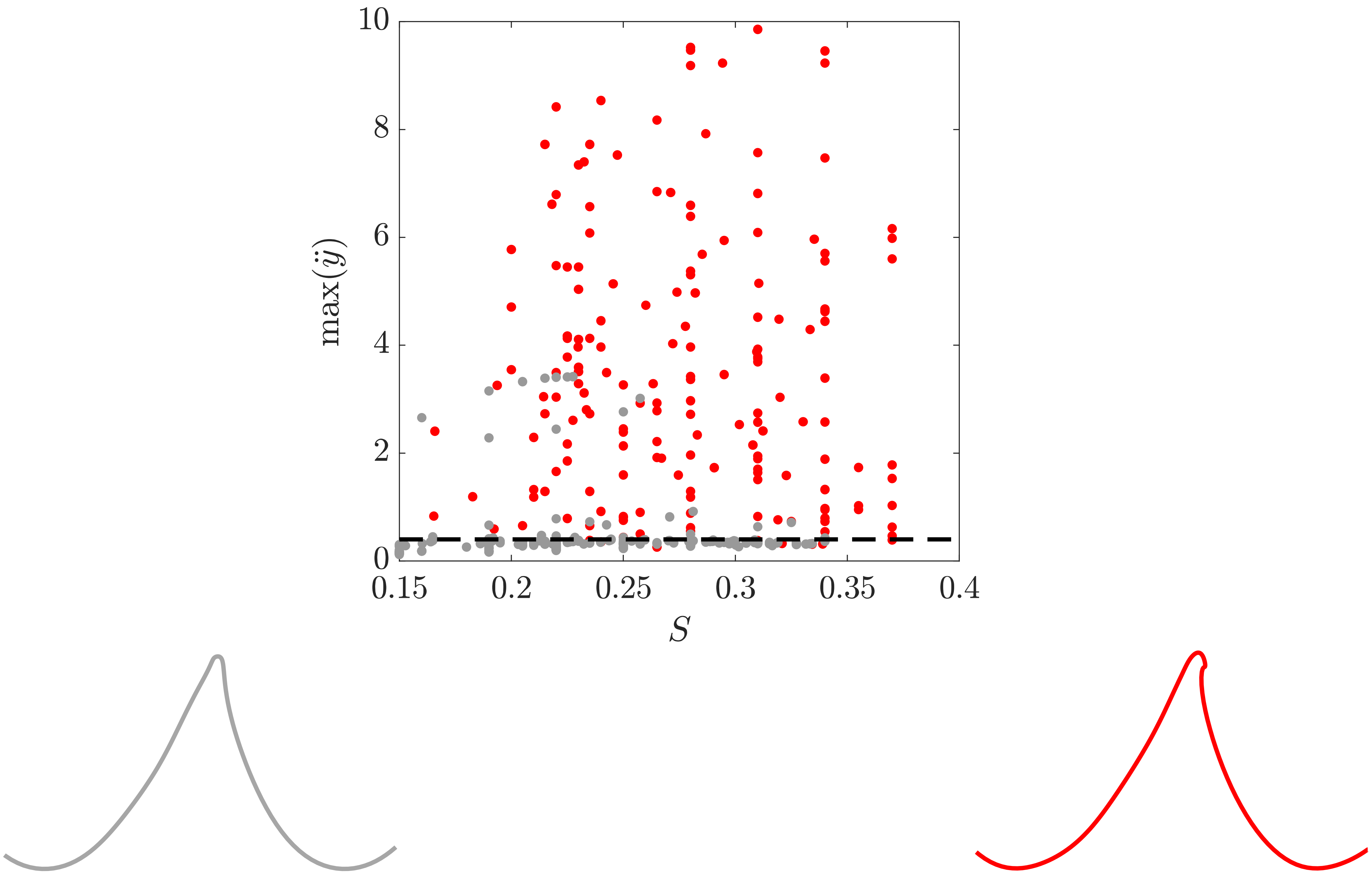
References:

- Craciunescu, C. C., & Christou, M. (2020). On the calculation of wavenumber from measured time traces. *Applied Ocean Research*, *98*, 102115.
- Pizzo, N., Murray, E., Smith, D. L., & Lenain, L. (2021). The role of bandwidth in setting the breaking slope threshold of deep-water focusing wave packets. *Physics of Fluids*, *33*(11), 111706.
- Wu, C. H., & Yao, A. (2004). Laboratory measurements of limiting freak waves on currents. *Journal of Geophysical Research: Oceans*, *109*(C12).

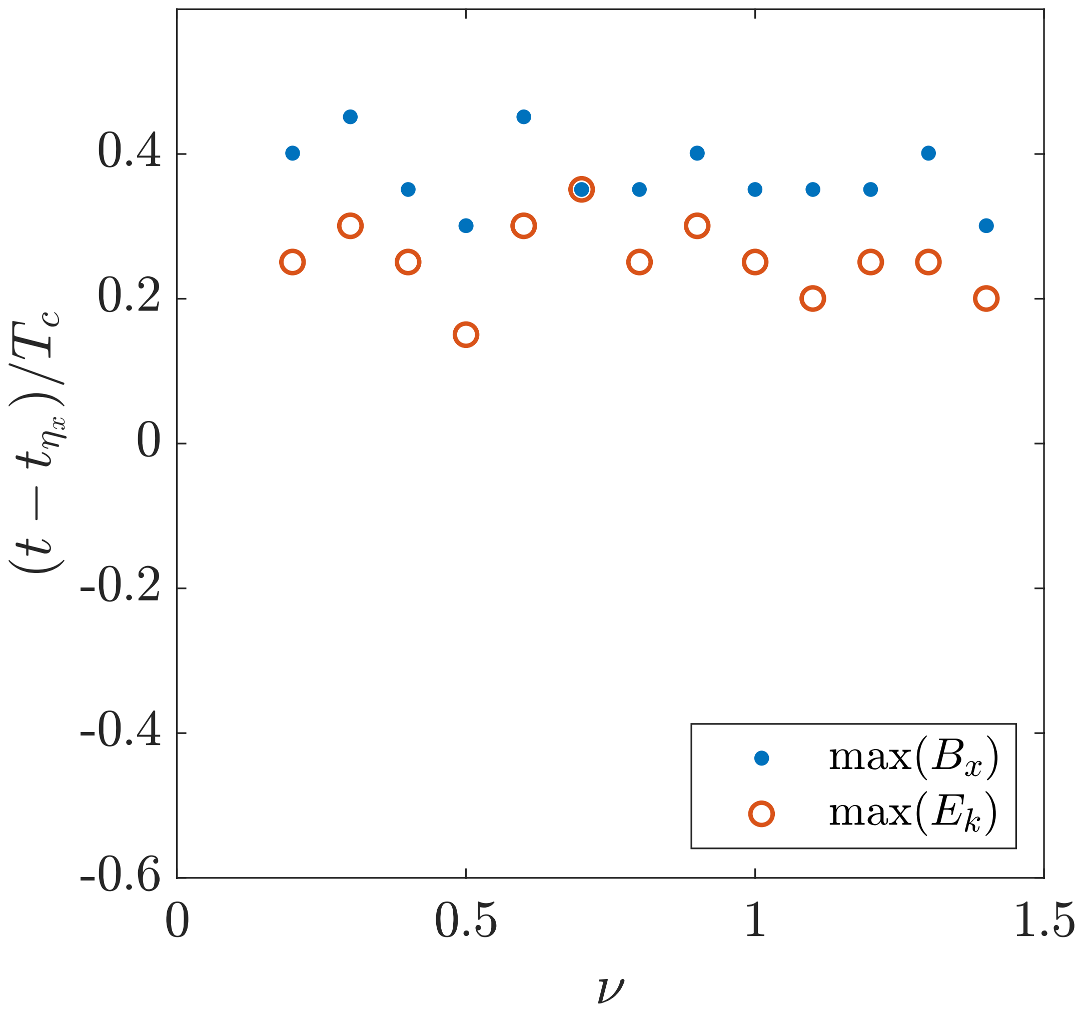
- Please email me for list of references from comparison figure

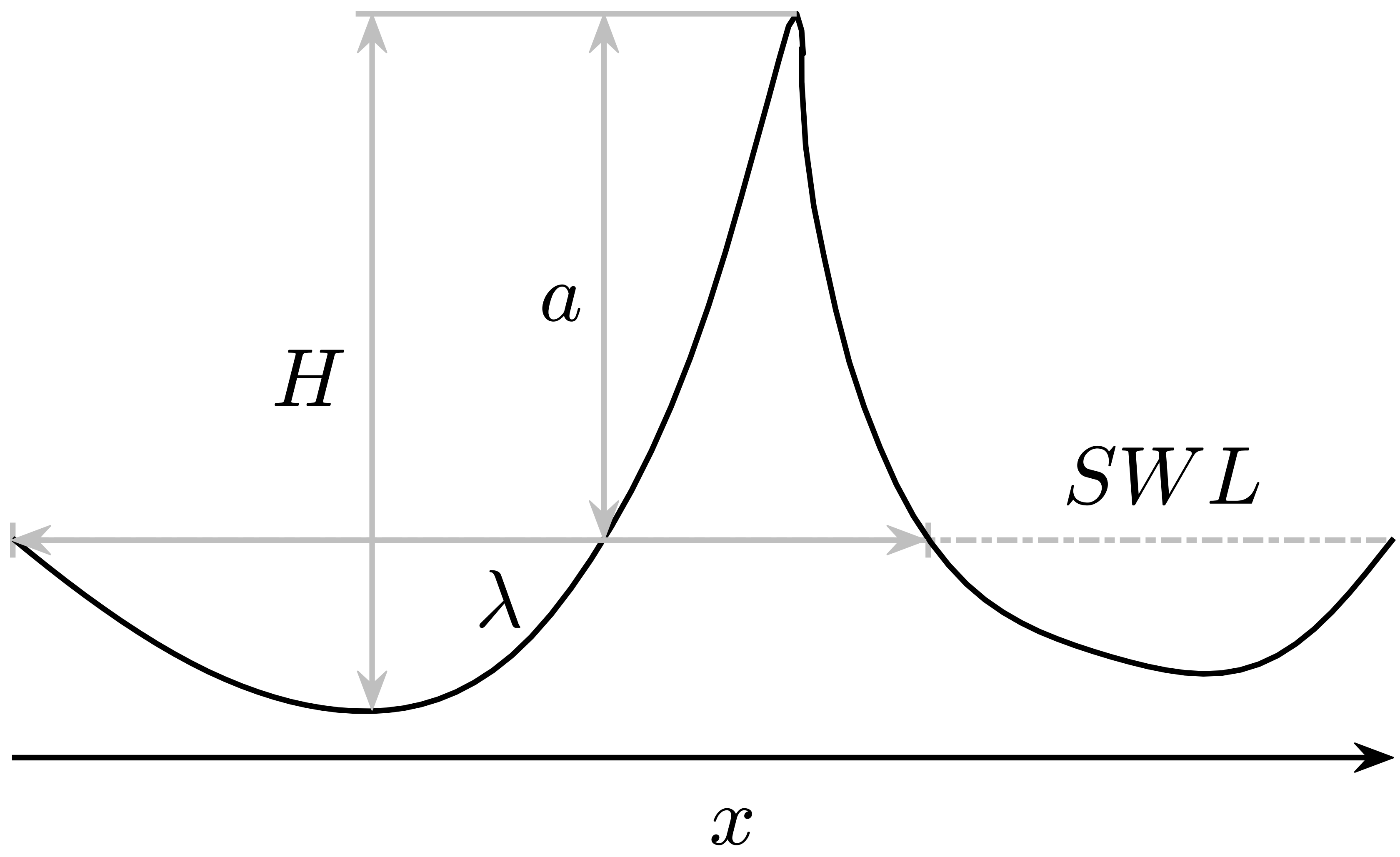


x (pix)
800
1000
1200
y (pix)
1500
2000
2500
3000
3500
0.5
0
-0.5
η_x
-200
-100
0
100
200
300
400
x (mm)
t_1
t_2
t_3

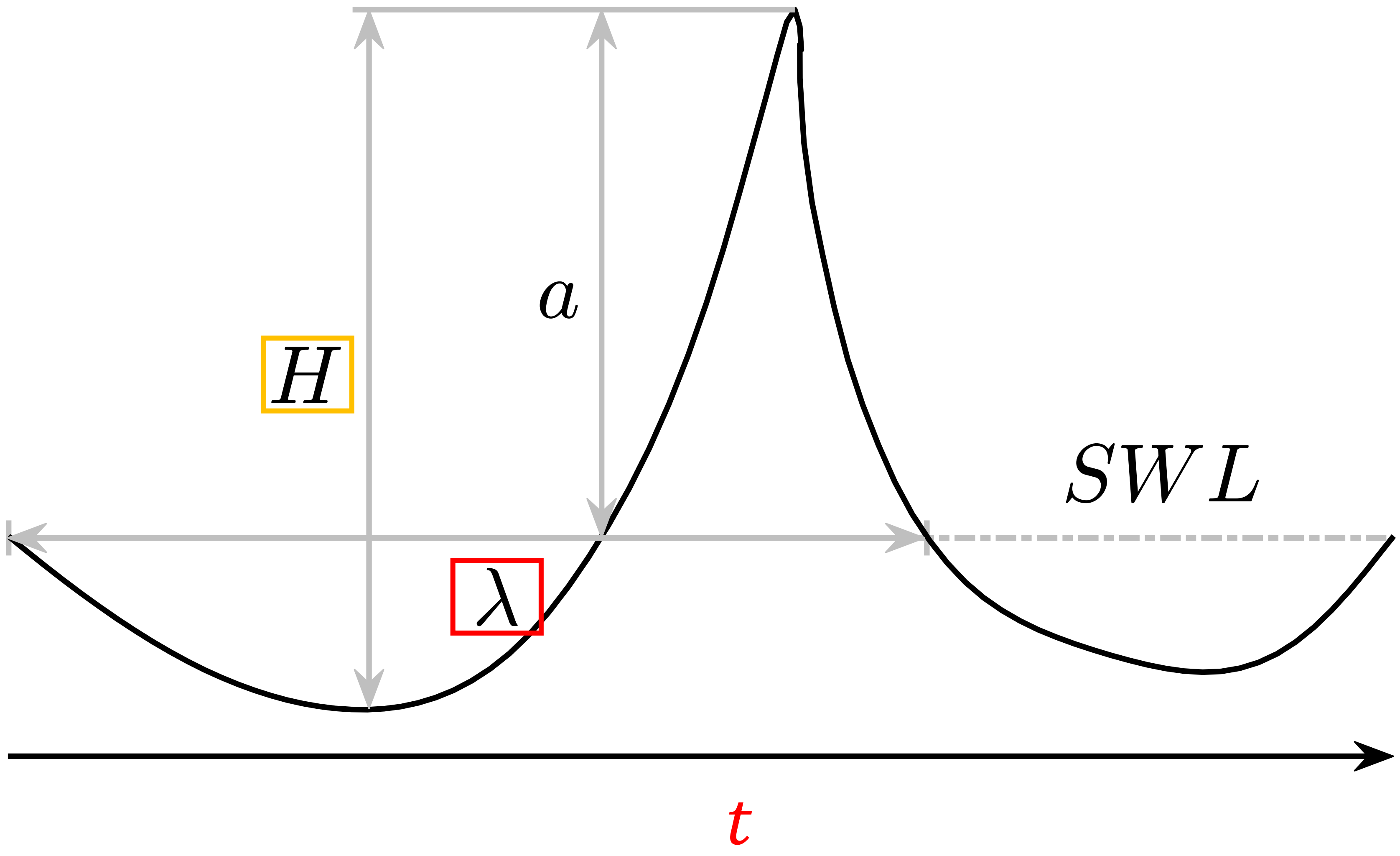


10
8
6
4
2
0
0.15
0.2
0.25
0.3
0.35
0.4
$\max(\ddot{y})$
S

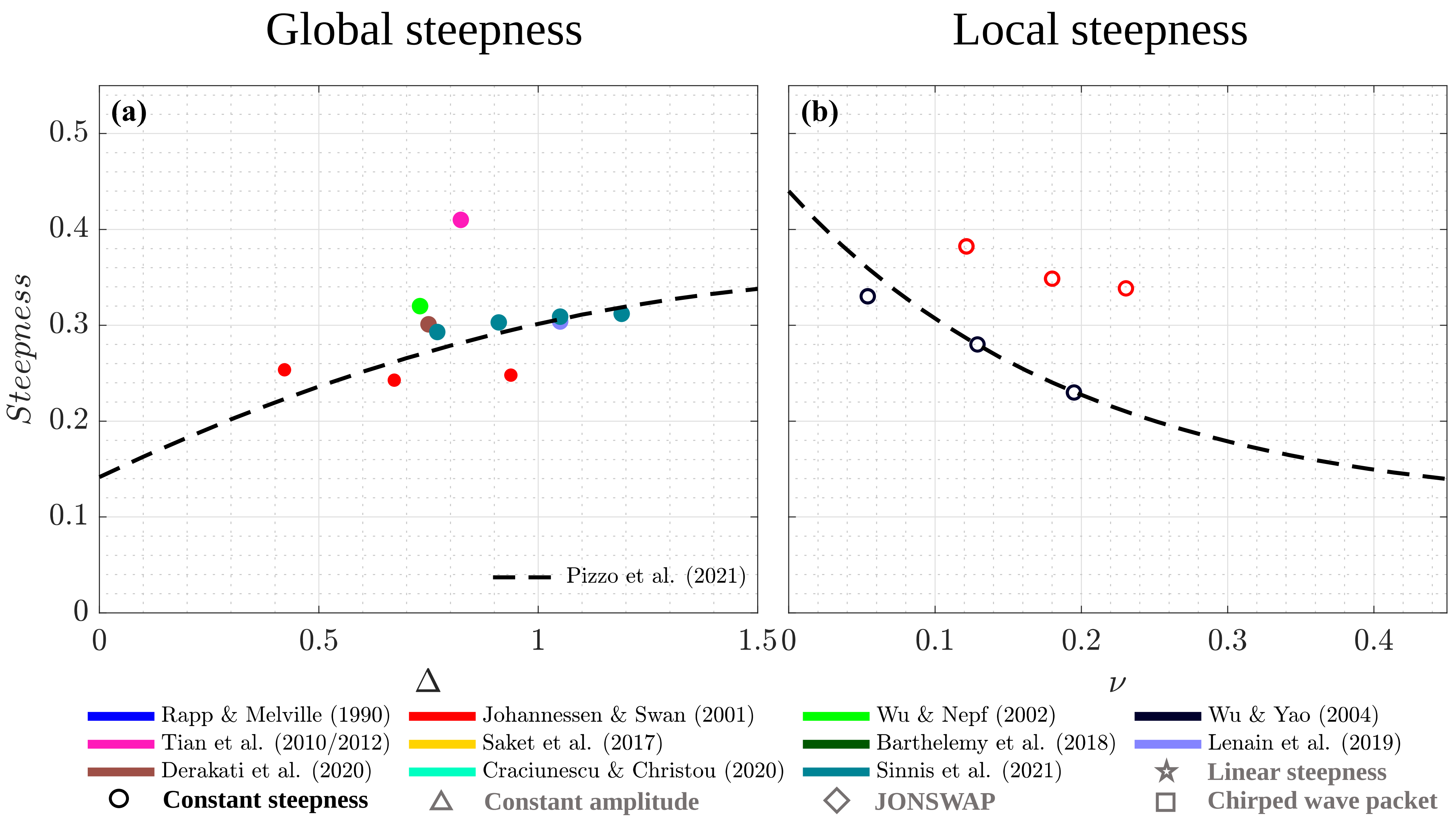

0.4
0.2
0
-0.2
-0.4
-0.6
0
0.5
1
1.5
ν
$(t - t_{\eta_x})/T_c$
$\max(B_x)$
$\max(E_k)$



a
H
SWL
λ
x



Craciunescu, & Christou (2020)


Global steepness
Local steepness
(a)
(b)
Steepness
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
0
0.1
0.2
0.3
0.4
Δ
ν
Pizzo et al. (2021)
Rapp & Melville (1990)
Johannessen & Swan (2001)
Wu & Nepf (2002)
Wu & Yao (2004)
Tian et al. (2010/2012)
Saket et al. (2017)
Barthelemy et al. (2018)
Lenain et al. (2019)
Derakati et al. (2020)
Craciunescu & Christou (2020)
Sinnis et al. (2021)
Linear steepness
Constant steepness
Constant amplitude
JONSWAP
Chirped wave packet

$$\eta^{(1)}(x,t) = \sum_{n=1}^{N} a_n \cos(\theta_n)\,, \qquad \phi^{(1)}(x,z,t) = \sum_{n=1}^{N} a_n \frac{\omega_n}{k_n} e^{k_n z} \sin(\theta_n)$$

$$\text{where} \quad \theta_n = k_{nx} - \omega t + \varphi_n, \qquad and\ \theta_n = 0\ at\ t_0\ and\ x_0$$

$$S = \sum_{n=1}^{N} a_n\, k_n = 1$$

$$k_c = \frac{\sum_{n=1}^{N} a_n k_n}{\sum_{n=1}^{N} a_n} = \frac{(2\pi)^2}{g}$$

$$h = \infty, \qquad \omega_c = 2\pi, \qquad f_c = 1, \qquad a_0 = \sum_{n=1}^{N} a_n = \frac{g}{(2\pi)^2}$$

$$At\ x = 0\ and\ t = 0$$

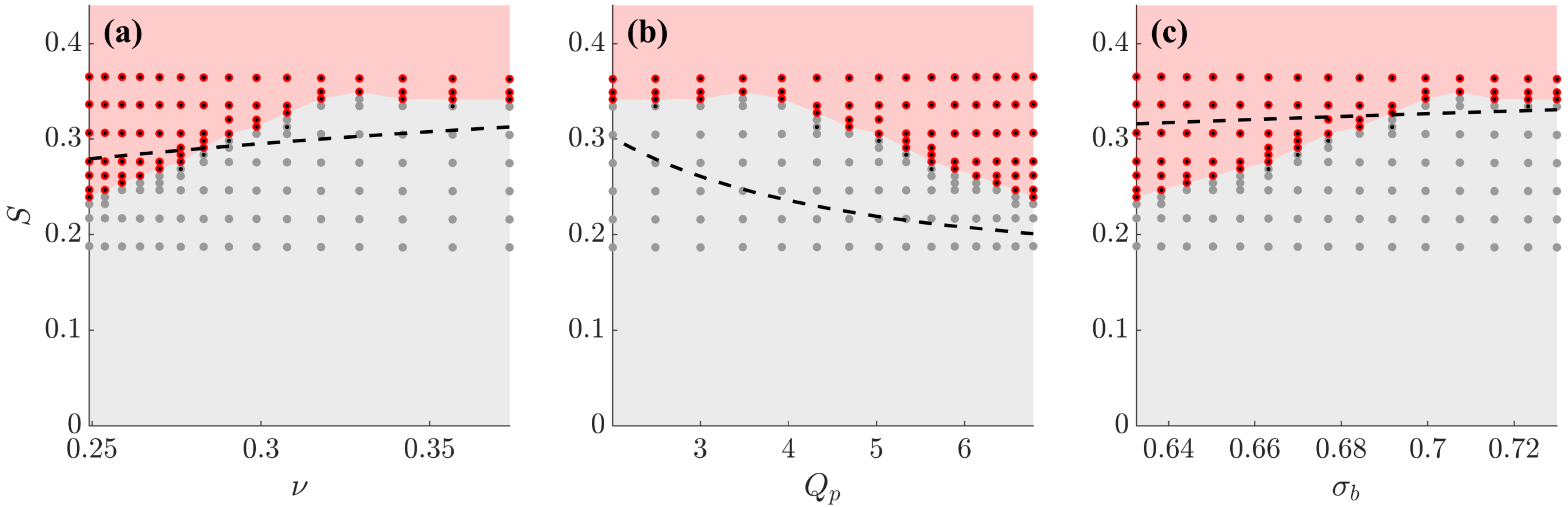
$$Crest\ velocity, \qquad C^{(1)} = \frac{\sum_{n=1}^{N} a_n\, \omega_n k_n}{\sum_{n=1}^{N} a_n k_n^2}$$

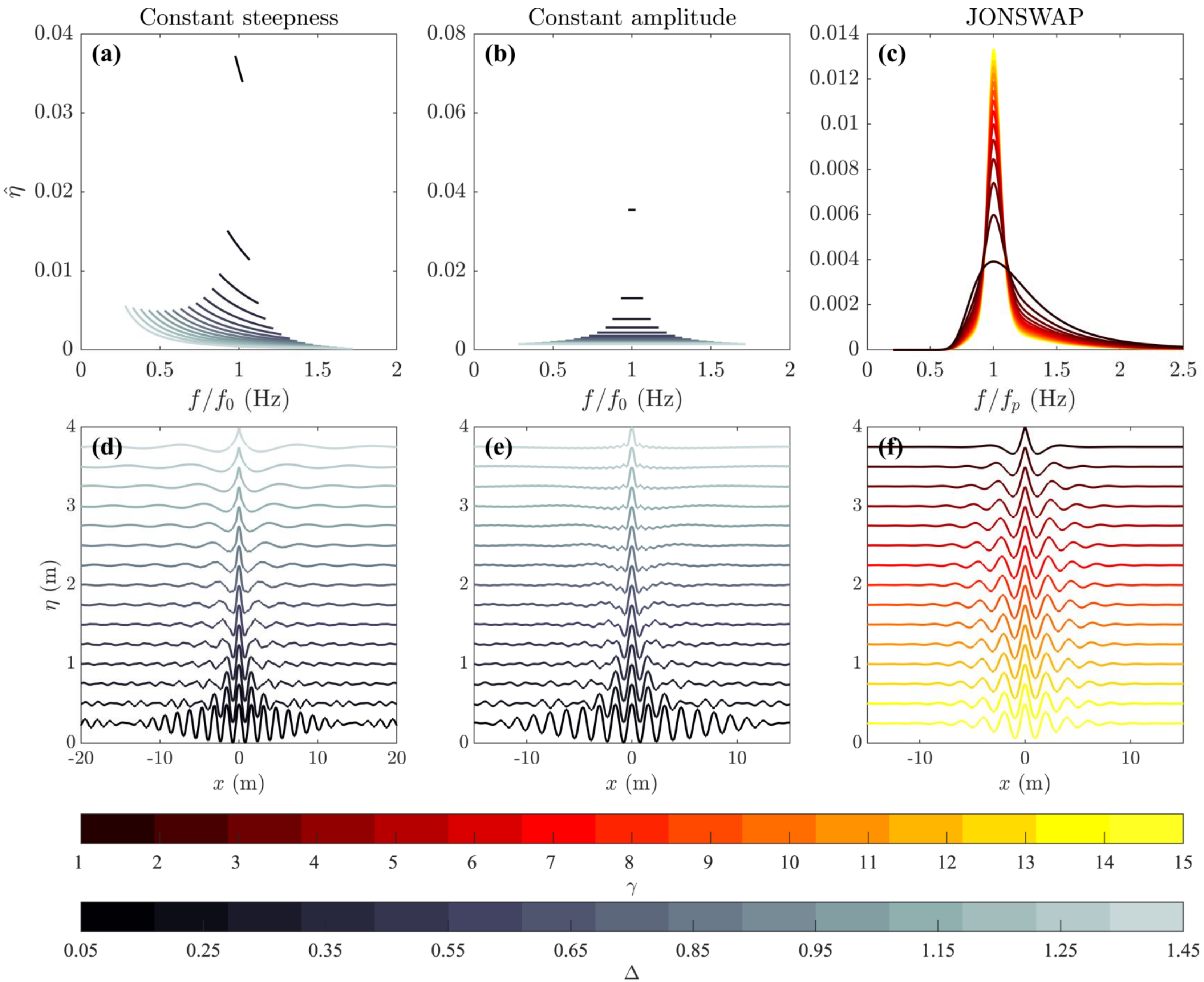
$$and\ at\ z = 0$$

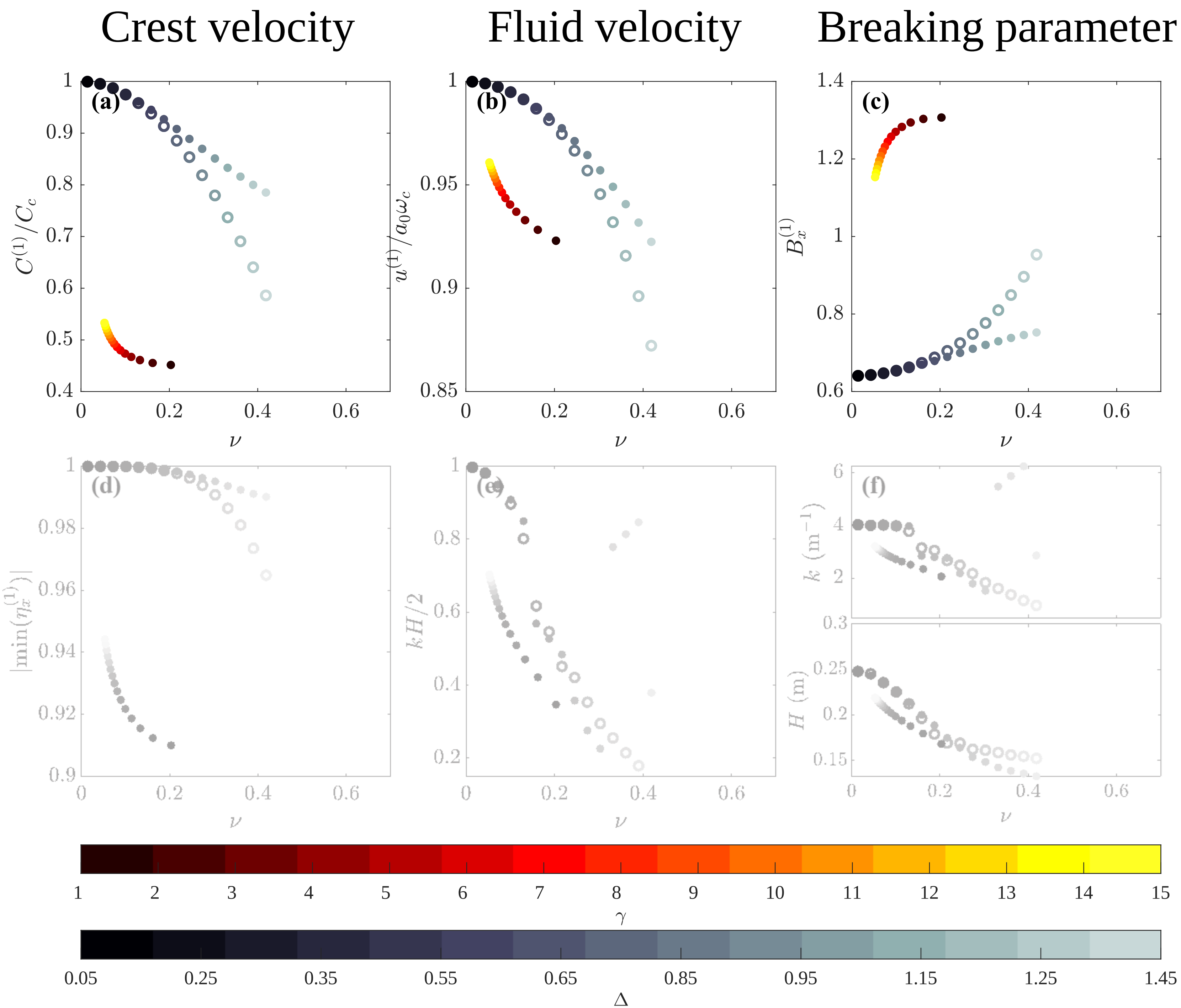
$$Fluid\ velocity, \qquad u^{(1)} = \sum_{n=1}^{N} a_n \omega_n$$

$$Breaking\ parameter, \qquad B_x^{(1)} = \frac{u^{(1)}}{C^{(1)}}$$

JONSWAP

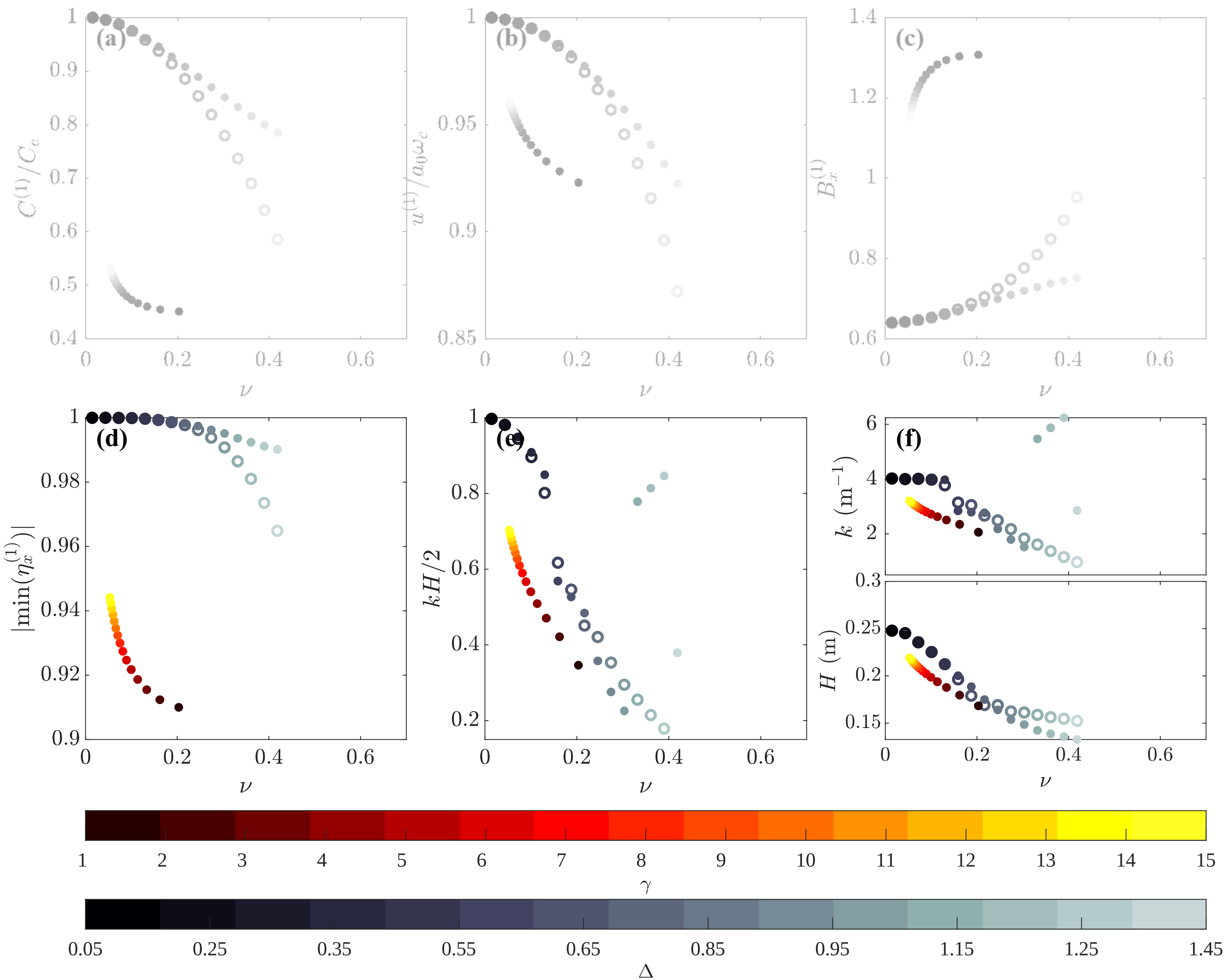


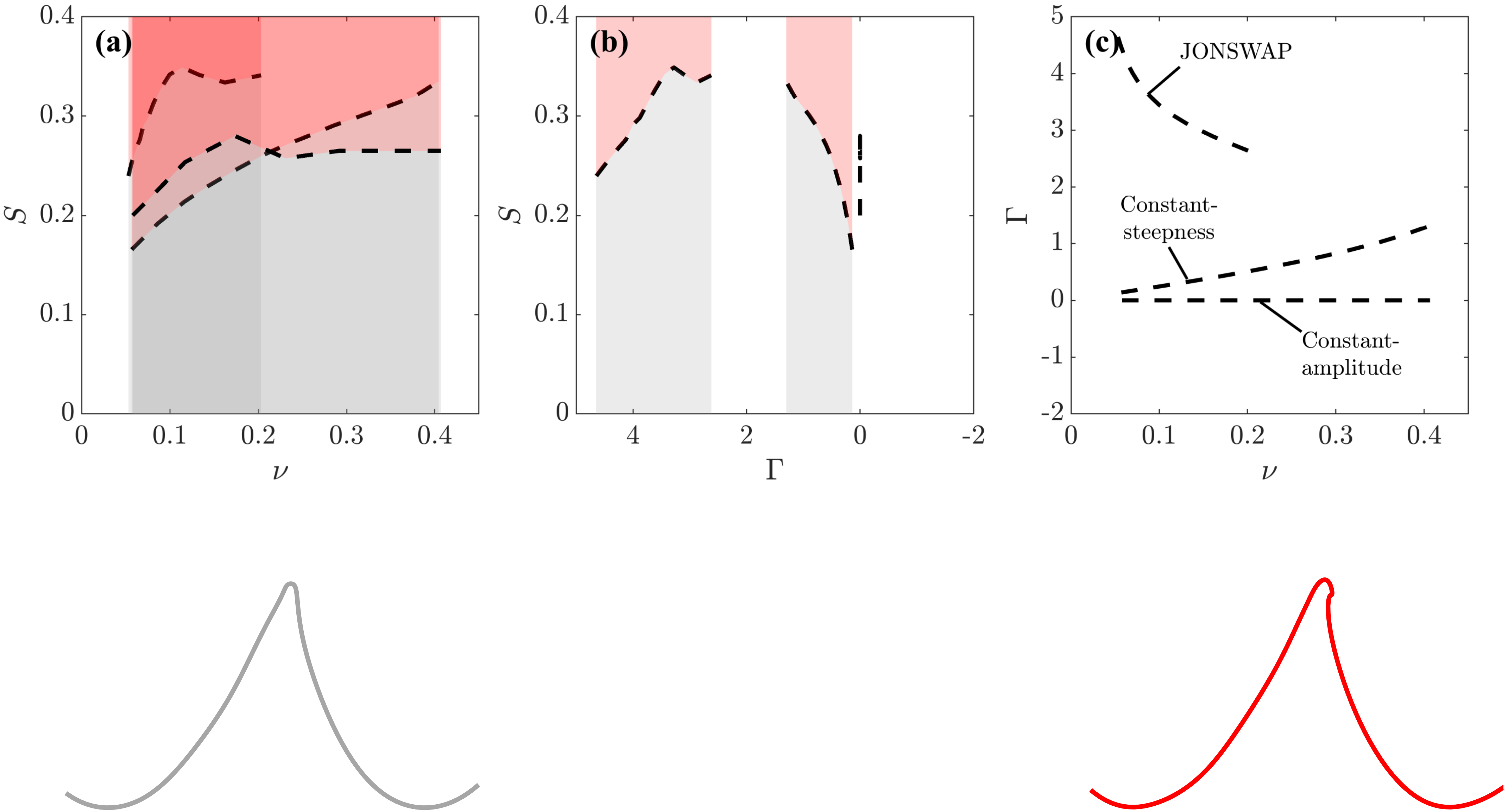

Constant steepness
Constant amplitude
JONSWAP
(a)
(b)
(c)
(d)
(e)
(f)
$\hat{\eta}$
f/f_0 (Hz)
f/f_0 (Hz)
f/f_p (Hz)
η (m)
x (m)
x (m)
x (m)
γ
Δ

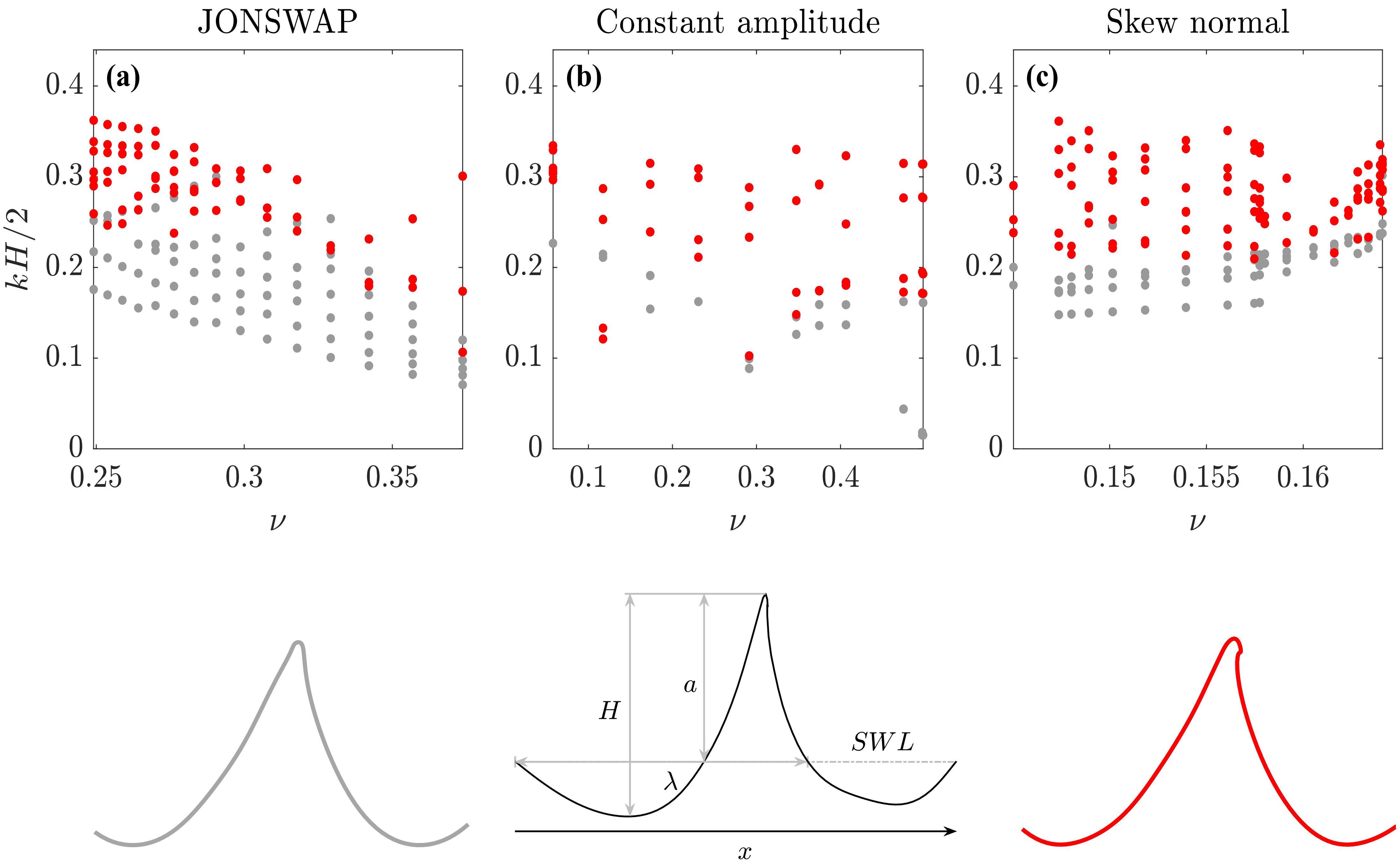


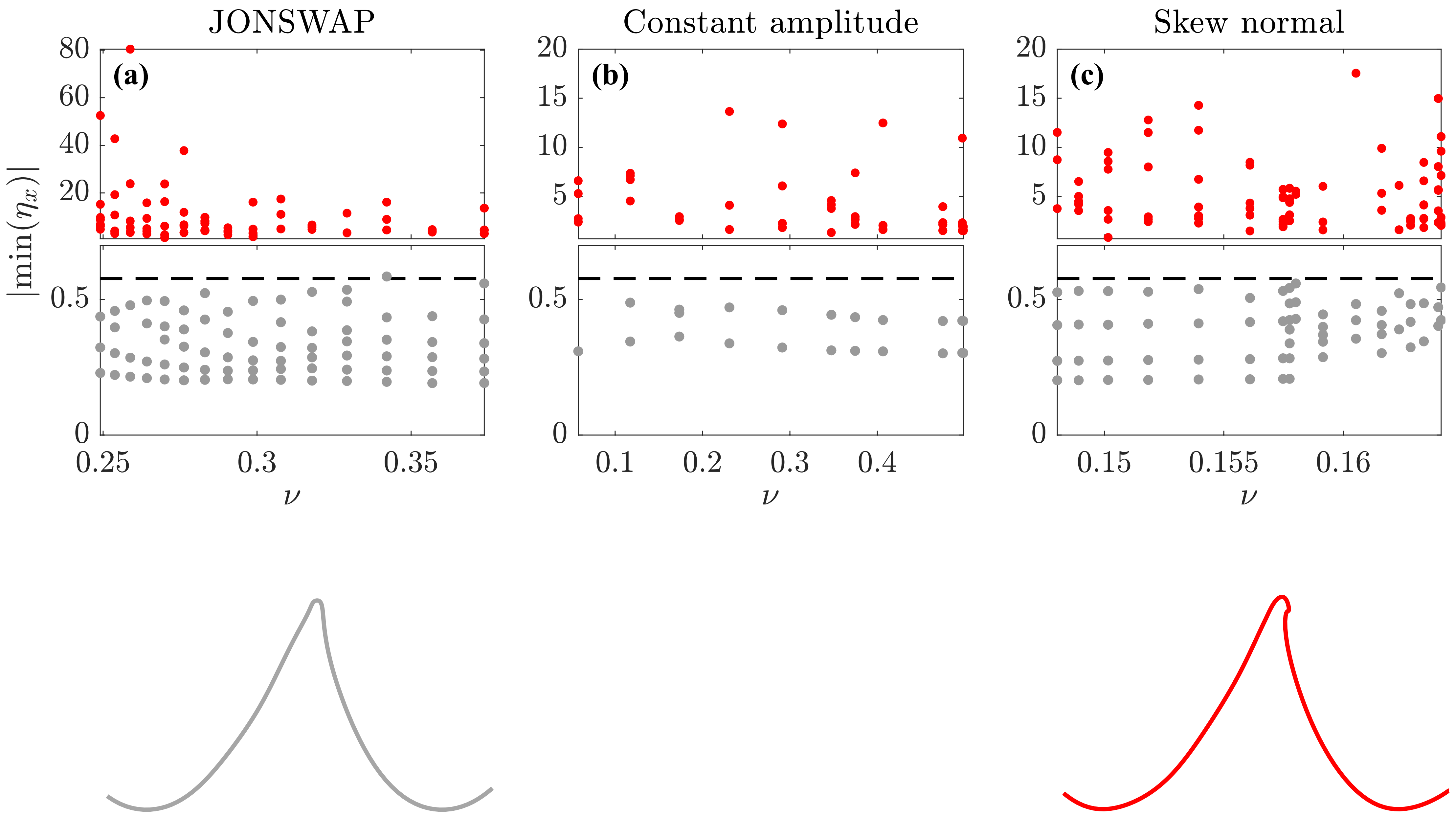
Crest velocity
Fluid velocity
Breaking parameter
(a)
(b)
(c)
(d)
(e)
(f)
$C^{(1)}/C_c$
$u^{(1)}/a_0\omega_c$
$B_x^{(1)}$
$|\min(\eta_x^{(1)})|$
$kH/2$
k (m^{-1})
H (m)
ν
γ
Δ

Local slope Local steepness Local k and H

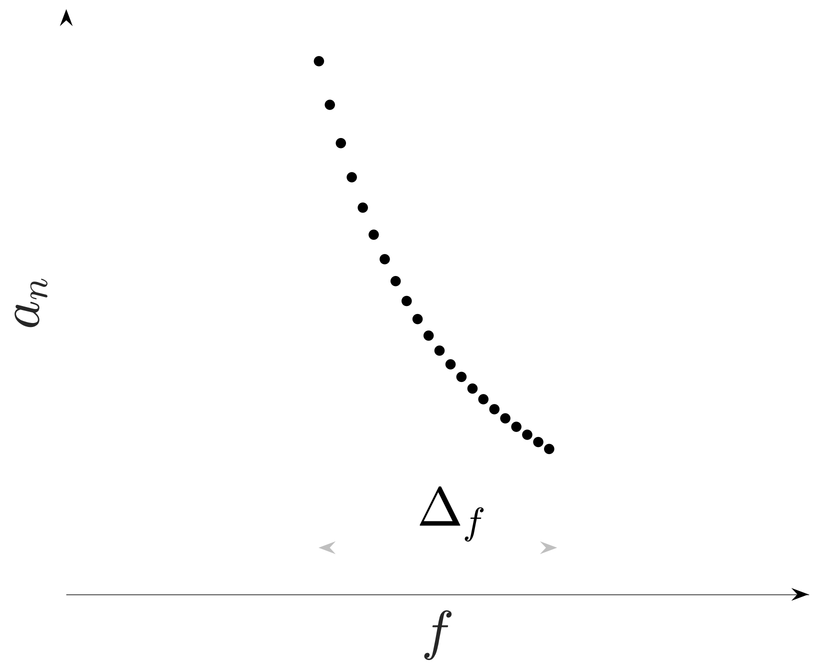



(a)
0.4
0.3
0.2
0.1
0
S
0
0.1
0.2
0.3
0.4
ν
(b)
0.4
0.3
0.2
0.1
0
S
4
2
0
-2
Γ
(c)
5
4
3
2
1
0
-1
-2
Γ
0
0.1
0.2
0.3
0.4
ν
JONSWAP
Constant-
steepness
Constant-
amplitude


JONSWAP
Constant amplitude
Skew normal
(a)
(b)
(c)
$kH/2$
ν
0
0.1
0.2
0.3
0.4
0.25
0.3
0.35
0.1
0.2
0.3
0.4
0.15
0.155
0.16
H
a
λ
SWL
x



JONSWAP
Constant amplitude
Skew normal
(a)
(b)
(c)
$|\min(\eta_x)|$
ν
80
60
40
20
0.5
0
15
10
5
0.25
0.3
0.35
0.1
0.2
0.4
0.15
0.155
0.16



$s_\star = 0.0579\Delta^2 + 0.2177\Delta + 0.1417$ Pizzo et al. (2021)

$$\delta S = 5 \times 10^{-4}$$